Estimating Resource Games with Nonlinear Dynamics

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October 24, 2026

Abstract

Many policy contexts, from fisheries management to epidemic control, are governed by nonlinear reproduction dynamics and by strategic interaction among multiple decision makers. This combination has historically made structural estimation computationally forbidding. We develop a tractable Generalized Method of Moments (GMM) framework for dynamic resource games with heterogeneous agents and fully nonlinear, interactive state dynamics. We provide sufficient conditions under which the Markov-perfect Nash equilibrium admits analytical optimality conditions, and establish consistency and asymptotic normality of the resulting GMM estimator. Applying the framework to the Japanese anchovy fishery shared by China, Japan, and Korea, we recover each country's implicit ecological valuation and simulate the equilibrium effect of climate-driven shifts in fish migration patterns. We further show that linearizing the equations of motion, the standard route to tractability, substantially understates these ecological values.

Keywords: stochastic dynamic game, structural estimation, GMM, renewable resource, fisheries

JEL Codes: Q20, C73, C57

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The two authors contributed equally to this work. We are grateful to Douglas Steigerwald, Christopher Costello, Andrew Plantinga, Ted Bergstrom, Gonzalo Vazquez-Bare, Antony Millner, Olivier Deschenes, Bruno Nkuiya, Max Farrell, Kyle Meng, Tong Li, conference participants at 2026 Western Economic Association International (WEAI) annual conference and 2026 Taiwan Economic Research for their insightful comments and suggestions.

1 Introduction

Structural estimation in fields like resource economics and public health is often computationally challenging due to two primary factors. First, the underlying systems are governed by *meaningful* nonlinearities in their equations of motion, such as biological reproduction rates or exponential disease transmission.[1] Linear approximations of equations of motion are often inadequate, as capturing nonlinear dynamics is essential for deriving accurate policy insights—such as preventing fish stock collapses (Costello et al., 2020) or implementing timely epidemic control (Atkeson, 2020; Toda, 2020; Acemoglu et al., 2020).[2] Second, these problems frequently involve strategic interactions among multiple players. These range from neighboring countries harvesting shared migratory fish to interconnected cities managing infectious diseases (Long, 2011; Huang and Smith, 2014; Fenichel et al., 2014; Tsao and Costello, 2024). At first glance, these complexities might suggest that tractable estimation is never feasible. This paper shows otherwise.

We develop a novel Generalized Method of Moments (GMM)-based estimation framework for a class of dynamic games with nonlinear equations of motion, which is particularly prevalent in fisheries, disease control, and common-pool resources. We provide a set of sufficient conditions under which our approach is feasible and establish its consistency and asymptotic normality. To illustrate the method's practical utility, we apply it to an anchovy harvesting game in East Asia. By recovering unobserved parameters regarding each country's ecological preferences, we demonstrate how this method provides new quantitative insights.

Despite recent theoretical breakthroughs in modeling dynamic resource games, tractable structural estimation methods for them remain vital (Costello et al., 2019; Tsao and Costello, 2024; Quaas and Winkler, 2025). While these theoretical advances provide new ways to characterize equilibrium behavior, sufficiently general models often yield ambiguous predictions, and policy insights can be highly sensitive to underlying structural parameters. Structural estimation in this literature has traditionally relied on computationally intensive numerical methods and context-specific restrictions on intertemporal behavior (Hotz and Miller, 1993; Bajari et al., 2007; Huang and Smith, 2014).

This paper addresses three research questions. We first ask whether a tractable estimation strategy can be developed for stochastic dynamic games with nonlinear, interdependent equations of motion. We then investigate how restrictive the conditions for our approach actually are. Finally, we explore the empirical and theoretical insights this approach can yield for resource economics and applied dynamic games.

[1] We focus on nonlinearities in the equations of motion rather than in the payoff function. Linear-quadratic games, for example, allow for nonlinear payoffs but assume linear equations of motion, which is not what we study in this paper.

[2] Section 4.5 demonstrates that estimation based on linearized equations of motion can be significantly biased in our context.

First, we establish sufficient conditions for GMM estimation in stochastic dynamic games with nonlinear equations of motion. Generalizing recent work in dynamic resource games (Costello et al., 2017, Tsao and Costello, 2024), we provide conditions that yield an analytical characterization of the Markov-perfect Nash equilibrium (MPNE). That is, the MPNE is defined by optimality conditions composed of known primitives and functions, removing the dependency on non-explicit value functions. These analytical optimality conditions generate overidentified moment conditions for the model's structural parameters, resulting in a GMM estimator that is consistent and asymptotically normal under standard regularity conditions. Overall, our framework accommodates strategic behaviors under nonlinear dynamics while maintaining computational tractability.

Second, we evaluate the generality of these sufficient conditions to demonstrate that they are transparent rather than overly restrictive. The conditions are: (1) Markov shocks, (2) shock exogeneity with no feedback, (3) optimal escapement as the control variable, and (4) no state-control interaction in payoff functions. The latter two are particularly important. The third condition implies that equilibrium behavior involves choosing the "optimal escapement" of variables of interest, which is standard in many stock management settings. For example, in fisheries, choosing harvest quantity is equivalent to choosing the stock after harvest (Weitzman, 2002).[3] The fourth condition simplifies estimation while still permitting "stock effects" in payoffs (Costello et al., 2017; Tsao and Costello, 2024).[4] We demonstrate that migratory fisheries, epidemic control, and invasive species management serve as examples that naturally satisfy these conditions.

Third, we show how our estimation approach can resolve empirical questions for which dynamic game theory alone delivers ambiguous predictions. We apply the method to the Japanese anchovy fishery shared by China, Japan, and Korea. This setting is particularly well suited to our framework for two reasons. First, Japanese anchovy migrate across the Exclusive Economic Zones (EEZs) of all three countries, yet the stock is not subject to formal cooperative management. Second, stock dynamics are nonlinear and strategically interdependent across countries. Using data from the RAM Legacy Database and Costello et al. (2016), we apply our GMM framework to estimate country-specific trade-offs between harvest revenue and ecological stock value (for example, as prey for other commercially valuable species). Based on these GMM estimates, we then simulate the impact of climate-driven northward migration into Korea on each country's equilibrium harvesting strategies (Ma et al., 2019, Liu et al., 2020).

Although rising anchovy inflows may imply larger harvests in Korea, the equilibrium effect is *ambiguous*.

[3] Let x be the stock before harvest and h the harvest amount. Given x, choosing h is equivalent to choosing escapement $(x-h)$.
[4] See Section 3.1 for more details about the stock effects.

Two opposing forces exist: while higher inflow allows Korea to harvest more, the increased outflow from China and Japan incentivizes these two countries to harvest more intensely before the fish migrate, which in turn encourages Korea to reduce harvest to maintain local stock for its ecological value. Our empirical results show that while both channels exist, the latter dominates. Surprisingly, in the new steady state, Korea maintains a greater equilibrium stock but realizes *less harvest*. This result highlights the necessity of empirical estimation when theory prediction alone remains inconclusive.

Finally, we compare our GMM estimates of ecological stock value parameters with estimates obtained by linearizing the equations of motion around the steady state. We find that the linearization approach can severely underestimate stock values and may even produce implausible negative estimates. This result underscores the importance of accounting for nonlinear dynamics in resource games and illustrates the usefulness of our GMM framework.

We make two primary contributions. First, we contribute to the methodological literature on the structural estimation of dynamic games. Although GMM is widely used in single-agent dynamic optimization (Hansen and Singleton, 1982; Costello and Plantinga, 2023), it has proven difficult to extend to games. In single-agent problems, the dynamic envelope theorem yields moment conditions (Euler equations) written in terms of observable states and controls (Acemoglu, 2008). In dynamic games, an agent's continuation value depends on *rivals' strategic choices* through the evolution of future states, so the envelope theorem alone does not deliver analytical optimality conditions.[5] We show that this obstacle can be overcome by exploiting two common features of resource economics: optimal escapement as the control variable (Assumption 3) and no state-control interaction in payoffs (Assumption 4).

The restrictions we chose distinguish our estimator from the standard methods applied to dynamic games. Compared to simulation-based approaches, such as the method of simulated moments (MSM) (McFadden, 1989; Duffie and Singleton, 1990) and two-step estimators (Bajari et al., 2007), our method avoids the need to repeatedly solve or forward-simulate the equilibrium. Those simulation-based approaches require strict parametric assumptions about the shock distribution in order to draw from it. In contrast, because our moment conditions are analytical functions of observables and known primitives, the equilibrium is never simulated during estimation, and no distributional assumptions are needed. Alternatively, conditional choice probability (CCP) methods (Hotz and Miller, 1993; Aguirregabiria and Mira, 2007) avoid solving the model, but they rely on a first-stage estimate of choice probabilities. This requires a discrete action space and a large dataset. Under our specific restrictions, we avoid the need for a first stage entirely, allowing us to

[5] See Remark 1 for a proof.

accommodate continuous states and controls. While we do not claim universal dominance over these broader methods, when a game fits our assumptions, our estimator yields a dramatic reduction in both computational and data requirements.

Second, we contribute to empirical resource economics by developing an easy-to-implement framework for quantifying strategic behavior in games with nonlinear dynamics. Existing approaches face a difficult tradeoff. Linearizing the equations of motion yields a tractable procedure but can generate substantively biased estimates when the core economic insight hinges on the nonlinearity. Estimating a full nonlinear model is computationally challenging and requires context-specific assumptions, such as modeling only within-season growth to bypass inter-seasonal dynamics (Huang and Smith, 2014). Our framework occupies a middle ground, preserving the full dynamic structure while facilitating a tractable GMM estimator.

More broadly, structural estimation is valuable because theory alone rarely yields unambiguous policy prescriptions. As our application illustrates, theoretical characterizations of complex dynamic games frequently yield predictions that depend entirely on unobserved parameters. Our framework provides a tractable path for taking these models to the data, and the assumptions fit well within resource economics and other contexts featuring nonlinear dynamics, such as epidemic control or the management of invasive species.

The remainder of the paper is structured as follows. Section 2 develops the model. Section 3 presents a set of sufficient conditions for our estimation framework and derives the GMM estimator with desirable asymptotic properties. Section 4 applies the estimation method to a Japanese Anchovy harvesting game in Asia. Section 5 concludes. Proofs and additional analysis are in the Appendix.

2 The Dynamic Game

2.1 Importance of Nonlinear Dynamics

Nonlinear equations of motion fundamentally shape policy insights in resource economics and public health, rather than being mere technical inconveniences. For example, a fishery's logistic reproduction function determines its carrying capacity and subsequent harvest quotas, while exponential disease transmission dictates the timing of early lockdown decisions. Figure 1 illustrates these nonlinear dynamics.

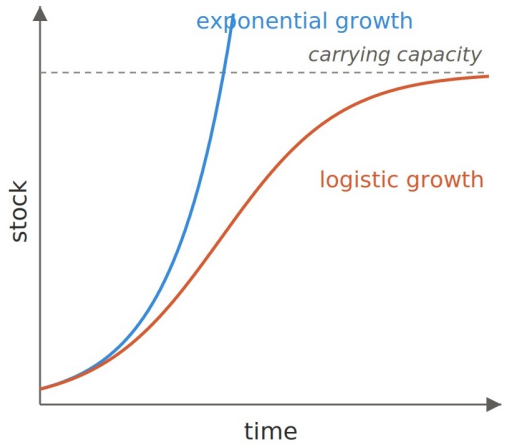


Figure 1: Illustration of Nonlinear Dynamics.
Notes: This figure illustrates two types of nonlinear growth dynamics for a stock over time. The blue curve illustrates exponential growth, which increases continuously without bound. The orange curve demonstrates logistic growth, which initially increases rapidly but eventually levels off as the stock approaches the system's carrying capacity, indicated by the horizontal dashed line.

Estimating dynamic games involving these nonlinear ecological dynamics presents significant methodological challenges. Consider a dynamic game with two players, A and B, where each player's Bellman equation takes the following form:

$$V_{At}(\cdot) = \max_{\text{A's action}} \{\underbrace{\text{A's current payoff}}_{(1)} + \underbrace{\text{A's expected continuation value}}_{(2)}\} \tag{1}$$

$$s.t. \quad \underbrace{\text{nonlinear equations of motion shared by A and B}}_{(3)}$$

The primary estimation hurdle is that the first-order condition of this Bellman equation generally lacks an analytical solution when the equations of motion are nonlinear. Simply linearizing these equations is inadequate, as nonlinear dynamics are the most critical element of these problems. We will demonstrate in an application that this is not merely a hypothetical concern: linearization can severely bias the estimation and even yield incorrect sign.

If the equations of motion must remain nonlinear to capture the reality, the model must impose restrictions elsewhere to enable tractable estimation. Our position is that the restrictions should be imposed on the payoff function and shock process (elements (1) and (2) in Equation 1) rather than on the dynamics (element (3)). The remainder of this section formalizes the dynamic game and establishes sufficient conditions that are reasonable within resource economics and enable GMM estimation.

2.2 Model Setup

We consider a generic stochastic dynamic game where N heterogeneous players face nonlinear and interactive equations of motion that describe the dynamics.[6] The state variables are $\mathbf{x}_t = [x_{1t}, ..., x_{Nt}] \in \mathbb{R}^N$, representing the current state of each agent i in period t (e.g., fish stock).[7] Additionally, stochastic shocks $\mathbf{z}_t = [z_{1t}, ..., z_{Nt}] \in \mathbb{R}^N$ are realized at the beginning of each period t (e.g., oceanographic shocks affecting fish reproduction).[8] Therefore, the tuple $(\mathbf{x}_t, \mathbf{z}_t)$ summarizes the publicly known state in period t.

After observing $(\mathbf{x}_t, \mathbf{z}_t)$, agents make simultaneous, noncooperative decisions. Let each agent i's control variable in period t be $u_{it} \in \mathbb{R}$. For instance, u_{it} could represent the harvest quota set by country i's EEZ for migratory fish in year t. Together, these controls form the vector $\mathbf{u}_t = [u_{1t}, ..., u_{Nt}] \in \mathbb{R}^N$.

A system of N nonlinear equations of motion governs the underlying dynamics, such as fish reproduction or disease transmission:

$$x_{it+1} = g_i(\mathbf{x}_t, \mathbf{u}_t, z_{it+1}) \tag{2}$$

for $i = 1, ..., N$. This interdependent dynamic structure reflects a core feature of common resource games: no single agent can fully dictate their next-period state. For example, in fisheries, $g_i(\cdot)$ captures stock growth and cross-border migration, meaning x_{it+1} depends on the actions of all players ($\mathbf{u}_t$). This interdependence prevents the direct application of traditional Generalized Method of Moments (GMM) approaches following Hansen and Singleton (1982).

The timing of the dynamic game is illustrated in Figure 2. At the beginning of period t, random shocks $\mathbf{z}_t$ are realized, which determine the current state $\mathbf{x}_t$ via Equation 2. After observing these shocks and states, each agent i simultaneously chooses u_{it} to maximize the present value of their lifetime payoff, before the game advances to period $t + 1$.

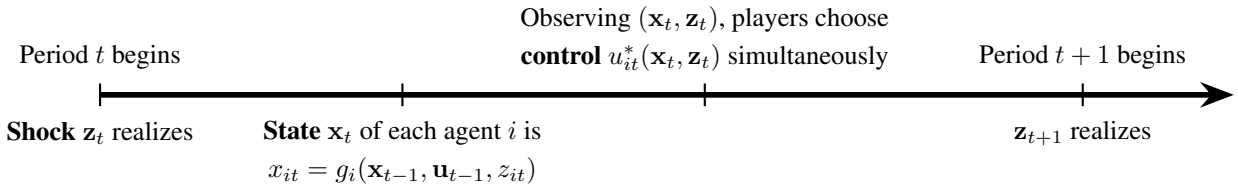


Figure 2: Sequence of Events within Each Period.
Notes: This timeline illustrates the sequence of events within each period of the stochastic dynamic game. Players independently and simultaneously choose u_{it}, and the resulting fixed point of best responses is the equilibrium.

[6] Examples include multiple spatially connected exclusive economic zone (EEZ) managers harvesting migratory fish, farmers extracting groundwater in a region, or countries managing mobile infectious diseases or invasive species across borders.

[7] For simplicity, we focus on $x_{it} \in \mathbb{R}$; however, x_{it} can also be a real vector in a more generalized model.

[8] Depending on the context, one can also assume that all agents experience the same shock. Namely, $z_{1t} = ... = z_{Nt}$ for all t.

Agent payoffs share a common functional form that is fully characterized by a K-dimensional structural parameters $\theta_i \in \mathbb{R}^K$. The values of θ_i may vary across agents to reflect heterogeneity. The current-period payoff function for agent i in period t is thus denoted as $f(x_{it}, u_{it}, z_{it}; \theta_i)$. Decision-making over an infinite horizon is governed by a discrete-time stochastic Bellman equation for each agent i:

$$V_{it}(\mathbf{x}_t, \mathbf{z}_t) = \max_{u_{it} \in \mathbb{R}} \{ f(x_{it}, u_{it}, z_{it}; \theta_i) + \delta \mathbb{E}[V_{it+1}(\mathbf{x}_{t+1}, \mathbf{z}_{t+1}) | \mathcal{I}_t] \} \tag{3}$$

$$s.t. \quad x_{it+1} = g_i(\mathbf{x}_t, \mathbf{u}_t, z_{it+1}) \quad \forall i = 1, ..., N$$

We adopt the Markov-perfect Nash equilibrium (MPNE) as our solution concept, where agents make simultaneous best-response decisions based solely on current state variables (Maskin and Tirole, 2001). Here, δ represents the discount factor and $\mathcal{I}_t$ is the information set at the beginning of period t.[9] The resulting equilibrium—the fixed point of these best responses—is defined as $\mathbf{u}_t^*(\mathbf{x}_t, \mathbf{z}_t) = [u_{1t}^*(\mathbf{x}_t, \mathbf{z}_t), ..., u_{Nt}^*(\mathbf{x}_t, \mathbf{z}_t)] \in \mathbb{R}^N$. MPNE is a standard equilibrium concept in resource economics, which avoids history-dependent strategies in favor of the simplest behavior consistent with rationality (Long, 2011; Costello et al., 2017).[10]

3 Structural Estimation

3.1 Sufficient Conditions for Deriving GMM Estimand

While deriving an analytical GMM estimand requires further assumptions, we argue the conditions we list are plausible within resource economics. We outline these assumptions below, followed by their underlying intuition and real-world examples.

Assumption 1 (Markov Property with Lag k)**.** *Assume that the shocks follow a Markov process with lag k, such that $p(\mathbf{z_t}|\mathbf{z_{t-1}}, \mathbf{z_{t-2}}, \dots) = p(\mathbf{z_t}|\mathbf{z_{t-1}}, \mathbf{z_{t-2}}, \dots, \mathbf{z_{t-k}})$ where p is the probability density function.*

Assumption 2 (Shock Exogeneity with No Feedback)**.** *Assume that for all t and $k = 0, 1, 2, ...$, $\frac{\partial \mathbf{z}_{t+k}}{\partial \mathbf{x}_t} = \mathbf{0}$ and $\frac{\partial \mathbf{z}_{t+k}}{\partial \mathbf{u}_t} = \mathbf{0}$.[11] In other words, current states and controls cannot affect current or future shocks, although current shocks can influence current and future states and controls.*

[9] A natural choice for $\mathcal{I}_t$ is $\sigma(\mathbf{x}_t, \mathbf{z}_t, \mathbf{x}_{t-1}, \mathbf{z}_{t-1}, \mathbf{u}_{t-1}, \cdots)$, the sigma algebra generated by the current state, current shock, and all past states, controls, and shocks.

[10] This equilibrium concept does not consider retaliation, trigger strategies, or other history-dependent commitments, as strategies depend only on the current state and not on past control choices of other agents.

[11] Here, $\mathbf{0}$ represents a matrix of zeros of the appropriate dimensions.

Assumption 3 (Escapement as Control)**.** *Assume that for all* $i = 1, ..., N$, $x_{it+1} = g_i(\mathbf{u}_t, z_{t+1})$.

Assumption 4 (No State-control Interaction in Current-period Payoff)**.** *Assume that for all* $i = 1, ..., N$, $\frac{\partial}{\partial x_{it}} \frac{\partial}{\partial u_{it}} f(x_{it}, u_{it}, z_{it}; \theta_i) = 0$.

Lemma 1 (State-independent MPNE)**.** *Under Assumptions 1 to 4, the interior MPNE* $\mathbf{u}_t^*(\mathbf{x}_t, \mathbf{z}_t)$ *simplifies to* $\mathbf{u}_t^*(\mathbf{z}_t)$*. That is, the optimal control rules* $\mathbf{u}^*$ *fluctuate with realized shocks* $\mathbf{z}_t$ *but remain entirely independent of the current states* $\mathbf{x}_t$.

Proof of Lemma 1: See Appendix. □

The seminal model by Weitzman (2002) exemplifies Lemma 1 and is a special case of our framework. The lemma's key insight is that optimal harvest strategies are defined by a target *escapement level* (the unharvested stock). Crucially, once we control for the serial correlation of shocks, this optimal escapement level is completely independent of the period's initial fish stock.

Theorem 1 (Analytical Optimality Conditions)**.** *Under Assumptions 1 to 4, the system of* N *optimality conditions has an analytical expression, where* $x_{it+1} = g_i(\mathbf{u}_t, z_{it+1})$ *and* $i = 1, ..., N$*:*

$$\frac{\partial f(x_{it}, u_{it}, z_{it}; \theta_i)}{\partial u_{it}} + \delta \mathbb{E}\left[\frac{\partial x_{it+1}}{\partial u_{it}} \frac{\partial f(x_{it+1}, u_{it+1}, z_{it+1}; \theta_i)}{\partial x_{it+1}} \middle| \mathcal{I}_t \right] = 0 \tag{4}$$

Proof of Theorem 1: See Appendix. □

Theorem 1 builds on the state-independence of Lemma 1 to demonstrate that the MPNE can be fully characterized by analytical optimality conditions. As shown in the subsequent section, these analytical expressions enable GMM estimation in a manner closely mirroring standard procedures for stochastic Euler equations (Hansen and Singleton, 1982; Costello and Plantinga, 2023).

We now outline the intuition behind these four assumptions, beginning with Assumption 1. Assumption 1 guarantees a fixed transition probability for current states conditional on past information, which is a prerequisite for the existence of a Markov-perfect Nash Equilibrium (MPNE). Shock processes such as independent and identically distributed (i.i.d.) draws or Vector Autoregressions (VAR) readily satisfy this requirement.[12]

Assumption 2 precludes feedback from current states or controls to future shocks, even as it permits serial correlation within the shock process itself. In a fisheries context, for example, an oceanographic shock

[12] While our subsequent derivations assume a lag of one for notational simplicity, the framework easily generalizes to k lags.

naturally influences current and future fish stocks as well as subsequent harvest quotas. However, this assumption precludes the biological impossibility of reverse feedback: a manager's harvest decisions cannot retroactively or proactively alter ocean weather patterns in the future.

While these first two shock-related assumptions are standard in structural estimation for resources like fisheries and forests (Weitzman, 2002; McGough et al., 2009; Sloggy et al., 2020; Kelsall et al., 2023), the remaining two represent relatively new structural restrictions. Assumption 3 simplifies the system's dynamics by rendering them independent of the period's initial state once the control is chosen. Inspired by Weitzman (2002) and Costello et al. (2015), this approach models the manager's action as effectively selecting the remaining stock (i.e., the *escapement*) rather than the raw harvest amount.[13] Consequently, next year's stock depends solely on the aggregate fish left in the water ($\mathbf{u}_t$) and the upcoming oceanographic shock z_{it+1}. This principle applies well to other resource contexts. In groundwater management, for instance, next period's aquifer level depends on the volume left unextracted and subsequent random refills, entirely independent of the aquifer's initial volume before pumping began.

Finally, Assumption 4 prohibits direct interaction between state and control variables in the current-period payoff function, a condition satisfied by several resource economics models. The first primary example involves common or public goods extraction, such as fisheries (Weitzman, 2002; Costello and Polasky, 2008, Costello et al., 2019). These models typically feature linear revenue and nonlinear harvest costs. Let x_{it} be the fish stock, u_{it} the escapement (making $x_{it} - u_{it}$ the harvest), p the landings price, and $c(s, \theta_i)$ the marginal harvest cost. The resulting payoff function cleanly separates as follows:

$$f(x_{it}, u_{it}, z_{it}; \theta_i) = p(x_{it} - u_{it}) - \int_{u_{it}}^{x_{it}} c(s, \theta_i) ds \tag{5}$$

Crucially, this separability **does not imply the absence of stock effects**. If marginal costs $c(s, \theta_i)$ decrease as s increases, a larger overall fish stock (x_{it}) lowers the marginal cost of catching a single fish ($\int_{x_{it}-1}^{x_{it}} c(s, \theta_i) ds$). This cost nonlinearity captures the classic stock effect, where greater abundance directly translates to cheaper harvesting.

Managing undesirable "bads"—such as infectious diseases or invasive species—provides a second primary example satisfying this payoff restriction. In these scenarios, policymakers choose an optimal escapement level to balance cumulative damage against management costs, accounting for the stock's mobility and renewability (Costello et al., 2017; Tsao and Costello, 2024). If x_{it} is the pest stock, u_{it} the unmanaged es-

[13] With this assumption, a natural restriction on the domain of u_{it} is $[0, x_{it}]$.

capement, $k(s, \theta_i)$ marginal damage, and $c(s, \theta_i)$ marginal management cost, the (negative) payoff function is expressed as follows:

$$f(x_{it}, u_{it}, z_{it}; \theta_i) = \int_0^{u_{it}} k(s, \theta_i) ds + \int_{u_{it}}^{x_{it}} c(s, \theta_i) ds \tag{6}$$

As in the fisheries case, this structure preserves the economic impact of stock scarcity. As the remaining stock of the "bad" (x_{it}) shrinks, the marginal cost of removing one more unit ($\int_{x_{it}-1}^{x_{it}} c(s, \theta_i) ds$) sharply increases, which reflects the difficulty of hunting down and eradicating the final few disease vectors or invasive pests in a region.

Although Assumptions 3 and 4 impose meaningful structural restrictions, they are indispensable for using the MPNE as a GMM estimand. Remark 1 formalizes this argument. The fundamental barrier is that agents lack full control over their next-period state variables. Because x_{it+1} depends on the aggregate choices of all players, the dynamic envelope theorem cannot effectively substitute the control variable u_{it} with the next-period state variable. This failure leaves the MPNE characterized by value functions with unknown functional forms. Consequently, these unknown functions prevent the optimality conditions from being used as direct estimands in GMM.

Remark 1 (Challenges in Estimating Dynamic Games)**.** *The dynamic envelope theorem alone is insufficient to derive analytical optimality conditions for this game if one relies solely on Assumptions 1 and 2.*

Proof of Remark 1: See Appendix. □

3.2 GMM Estimand and GMM Estimator

Building on the analytical optimality conditions derived above, we now turn to estimating the K-dimensional structural parameters $\{\theta_i\}_{i=1}^N$ that fully characterize each agent i's payoff function of each agent i. Specifically, we extend the method proposed by Hansen and Singleton (1982) to define the GMM estimand and construct the corresponding estimator. For notational convenience, we take $t = 1$ as the initial period.

Theorem 1 establishes the following population optimality condition for each agent i and period t:

$$\mathbb{E}\left[\frac{\partial f(x_{it}, u_{it}, z_{it}; \theta_i)}{\partial u_{it}} + \delta \frac{\partial x_{it+1}}{\partial u_{it}} \frac{\partial f(x_{it+1}, u_{it+1}, z_{it+1}; \theta_i)}{\partial x_{it+1}} \middle| \mathcal{I}_t\right] = 0 \tag{7}$$

Let

$$\mathbf{w}_t = (\mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{u}_t, \mathbf{u}_{t+1}, \mathbf{z}_t, \mathbf{z}_{t+1})$$

and let $r_i(\cdot;\theta_i)$ denote the expression inside the conditional expectation in equation 7. The optimality condition can then be written as:

$$\mathbb{E}\left[r_i(\mathbf{w}_t;\theta_i)|\mathcal{I}_t\right] = 0 \tag{8}$$

In our setup, current and past states and shocks, together with past controls, are public information. Thus,

$$\sigma(\mathbf{x}_1, \mathbf{z}_1, \mathbf{u}_1, \ldots \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{z}_{t-1}, \mathbf{x}_t, \mathbf{z}_t) \subseteq \mathcal{I}_t$$

Let

$$\mathbf{X}_t = (\mathbf{x}_1, \ldots, \mathbf{x}_t), \mathbf{U}_t = (\mathbf{u}_1, \ldots, \mathbf{u}_{t-1}), \mathbf{Z}_t = (\mathbf{z}_1, \ldots, \mathbf{z}_t)$$

denote the histories of states, controls, and shocks, respectively. Let $G_i(\mathbf{X}_t, \mathbf{U}_t, \mathbf{Z}_t)$ be an M-dimensional function of these histories, where $M > K$. Because $G_i(\mathbf{X}_t, \mathbf{U}_t, \mathbf{Z}_t)$ is measurable with respect to $\mathcal{I}_t$, it can be taken outside the conditional expectation. Applying the law of iterated expectations then yield the unconditional moment restrictions stated in the following lemma.

Lemma 2 (Instruments for Estimation)**.** *If* $\mathbb{E}[\|G_i(\mathbf{X}_t, \mathbf{U}_t, Z_t)\|] < \infty$ *and* $\mathbb{E}[|r_i(w_t;\theta_i)|] < \infty$*, we have* $\mathbb{E}[G_i(\mathbf{X}_t, \mathbf{U}_t, Z_t) r_i(w_t;\theta_i)] = 0$.

Proof of Lemma 2: See Appendix. □

Lemma 2 provides M population moment restrictions for the K-dimensional parameter θ_i, with $G_i(\mathbf{X}_t, \mathbf{U}_t, \mathbf{Z}_t)$ serving as the instrument vector. Since $M > K$, θ_i is overidentified and this leads to the following population GMM estimand for each agent i in period t.

$$\mathbb{E}[G_i(\mathbf{X}_t, \mathbf{U}_t, \mathbf{Z}_t) r_i(\mathbf{w}_t;\theta_i)]' W_i \mathbb{E}[G_i(\mathbf{X}_t, \mathbf{U}_t, \mathbf{Z}_t) r_i(\mathbf{w}_t;\theta_i)] = 0$$

where W_i is a fixed positive definite weighting matrix.

For estimation, we observe states, controls, and shocks over T periods, $\{\mathbf{x}_t, \mathbf{u}_t, \mathbf{z}_t\}_{t=1}^T$. The GMM estimator $\widehat{\theta}_i$ minimizes the sample counterpart of the population GMM estimand:

$$\widehat{\theta}_i = \arg\min_{\theta \in \Theta} \left[\frac{1}{T}\sum_{t=1}^{T} G_i(\mathbf{X}_t, \mathbf{U}_t, \mathbf{Z}_t) r_i(\mathbf{w}_t;\theta)\right]' \widehat{W}_{iT} \left[\frac{1}{T}\sum_{t=1}^{T} G_i(\mathbf{X}_t, \mathbf{U}_t, \mathbf{Z}_t) r_i(\mathbf{w}_t;\theta)\right] \tag{9}$$

where $\widehat{W}_{iT}$ is a sequence of random weighting matrices that converges in probability to W_i as $T \to \infty$.

In the Appendix A.5, we establish the consistency and asymptotic normality of the GMM estimator and provide a consistent estimator of its covariance matrix.

4 Application: Anchovy Harvesting Game in East Asia

4.1 Context

This section presents an empirical application of dynamic games characterized by crucial nonlinearity in the equations of motion to illustrate the applicability of our estimation method. We examine the harvesting game of the Japanese anchovy involving China, Japan, and the Republic of Korea. The Japanese anchovy fishery exhibits four distinctive characteristics that make it an ideal context for our proposed framework.

First, the Japanese anchovy is a highly migratory species. As discussed in Liu et al. (2020), the Japanese anchovy regularly traverses the Exclusive Economic Zone (EEZ) boundaries of the three countries (the Yellow Sea, East China Sea, Sea of Japan, and along the Pacific coasts of Japan) between its spawning, feeding, and wintering grounds.[14] This biological connectivity implies that harvesting activities within one EEZ inevitably influence stock abundance in neighboring jurisdictions.

Second, minimal harvesting cooperation or formal agreements for Japanese Anchovy have been established among the three countries. The absence of effective international coordination underscores the relevance of analyzing the fishery within a non-cooperative game-theoretic framework.

Third, the Japanese anchovy provides both economic harvest revenue and ecological stock value. The ecological stock value derives from the species' role as essential prey for other commercially valuable fish.[15] This dual role implies that EEZ managers may balance the present values of harvest revenues against the ecological benefits related to stock abundance when managing harvest efforts. Although harvest revenues are well-documented in the fisheries literature (Costello et al., 2016), the extent to which managers value the ecological contribution of the Japanese anchovy remains limitedly understood. This motivates the need to estimate structural parameters representing this ecological valuation.

[14] Source: Food and Agriculture Organization of the United Nations.

[15] Economically, Japanese anchovy are valuable for direct human consumption and aquaculture, serving as feed and fish meal. China, Japan, and the Republic of Korea harvest significant quantities each year. In 2023, annual anchovy harvests amounted to 632,576 metric tons in China, 114,200 metric tons in Japan, and 164,100 metric tons in the Republic of Korea (Source: Food and Agriculture Organization of the United Nations). Ecologically, existing fishery literature acknowledges that the Japanese anchovy serves as a keystone species linking lower and upper trophic levels in marine food webs (Liu et al., 2020; Zhao et al., 2008). It provides essential prey for approximately 40 species of commercially valuable fish, including Japanese Spanish mackerel, small yellow croaker, sea bass, chub mackerel, olive flounder, and largehead hairtail.

Finally, ecologists have recently found that climate change is altering the migratory patterns and spatial distribution of the Japanese anchovy (Lecomte et al., 2004; Ma et al., 2019). Liu et al. (2020) predict that the Japanese anchovy will shift northward in response to rising sea surface temperatures. This raises a natural economic question: how will changes in migratory patterns affect harvest outcomes within each country's EEZ? This question is complex, as changes in migratory patterns not only affect the spatial distribution of biomass but also alter the strategic incentives of EEZ managers under a non-cooperative harvesting framework.

Collectively, these features motivate the application of our estimation method to the Japanese anchovy harvesting game in East Asia. We first estimate the unobservable, implicit, country-specific structural parameters regarding the valuation of the Japanese anchovy stock. We then use the resulting estimates to evaluate how shifts in migratory patterns affect the equilibrium levels of harvest and escapement in each country.

4.2 The Harvesting Game

In this section, we formalize the harvest game for Japanese anchovy among China, Japan, and the Republic of Korea. This application belongs to the class of dynamic games with critical nonlinear dynamics examined in this paper. We model three players—the Exclusive Economic Zone (EEZ) managers for each country—who independently set harvest quotas in a non-cooperative framework. In each period t, each EEZ manager observes the stock levels and selects a quota h_{it} to maximize the present value of utility, given rational expectations regarding future shocks and as a best response to other EEZ managers. Following Weitzman (2002), we define the control variable as escapement ($e_{it} = x_{it} - h_{it}$), representing the residual stock remaining after harvest.[16]

Each EEZ manager's utility is derived from two sources: harvest profits and the ecological value of the standing stock. Consistent with the biodiversity modeling literature, we assume the utility function is additively separable in these components (Polasky et al., 2004). Specifically, the current-period utility for manager i is:

$$U(x_{it}, e_{it}) = (P - C)(x_{it} - e_{it}) + b(x_{it}; \theta_i) \tag{10}$$

where P is the constant unit price of harvest, C is the constant marginal cost of harvest, and $b(\cdot)$ is the stock value function.[17] We treat managers as price takers, given that Japanese anchovy is a globally traded

[16] Defining the control in terms of escapement rather than harvest simplifies the notation and often provides greater economic insight into optimal management strategies.

[17] While we assume marginal costs are symmetric across countries for parsimony, this assumption can be relaxed should more granular cost data become available.

commodity with close substitutes, such as Peruvian and European anchovy.

For the stock value component, we specify $b(x_{it};\theta_i) = \theta_i x_{it}^m$, where $0 < m < 1$ ensures that the function is strictly increasing and concave in stock level x_{it}. This functional form is motivated by the ecological role of the Japanese anchovy as a primary prey species for various commercially valuable predators. From the perspective of an EEZ manager, the anchovy stock's value stems largely from its capacity to support predator biomass. Consequently, we model the manager's ecological utility as a function of the supported predator biomass y_{it}, such that $b(x_{it};\theta_i) = f_i(y_{it})$. Following the empirical evidence in Hatton et al. (2015), the relationship between predator and prey biomass follows a power law with an exponent of approximately 0.75 (i.e., $y_{it} = x_{it}^{0.75}$). By setting $m = 0.75$, the parameter θ_i effectively captures the weight that manager i assigns to the ecological value of the anchovy stock and serves as the primary parameter for our estimation. This parameter θ_i is unobservable, reflects country-specific heterogeneity in stock value, and serves as the primary structural parameter of interest in our estimation.

Biological dynamics are governed by a generalized logistic recruitment function, a standard specification in fisheries economics (Costello et al., 2016). The stock in country i evolves according to:

$$x_{it+1} = z_{it+1} \sum_{j=1}^{3} D_{ji} \left(e_{jt} + \frac{\phi+1}{\phi} R_j e_{jt} (1 - (\frac{e_{jt}}{K_j})^{\phi}) \right) \quad \forall i = 1, ..., 3 \tag{11}$$

where K_j denotes the carrying capacity, R_j is the intrinsic growth rate, and ϕ is the curvature parameter (Costello et al., 2016). At the end of each period, a fraction D_{ii} of the recruited stock remains within country i's EEZ, while a fraction D_{ij} migrates to country j's EEZ. Furthermore, biological reproduction is subject to an exogenous, multiplicative shock z_{it+1}, which is realized at the beginning of period $t+1$. Following McGough et al. (2009), we assume this stochastic process is stationary and follows a Markov chain. This spatial and temporal structure implies that the transition mapping from escapement e_{jt} to the subsequent stock level x_{jt+1} is both nonlinear and strategically coupled, as the state in each EEZ depends on the escapement levels of all players.

Integrating these components, the harvest game is characterized by a system of Bellman equations. For each country $i \in \{1, 2, 3\}$, the value function $V_{it}(\mathbf{x}_t, \mathbf{z}_t)$ satisfies:[18]

$$V_{it}(\mathbf{x}_t, \mathbf{z}_t) = \max_{0 \le e_{it} \le x_{it}} \{ \underbrace{(P - C)(x_{it} - e_{it})}_{\text{harvest profit}} + \underbrace{\theta_i x_{it}^{0.75}}_{\text{stock value}} + \delta \mathbb{E}[V_{it+1}(\mathbf{x}_{t+1}, \mathbf{z}_{t+1}) | \mathcal{I}_t] \} \tag{12}$$

[18] The three countries considered are China, Japan, and the Republic of Korea. However, our framework is generalizable and can be applied to an arbitrary number of N players.

$$s.t. \quad x_{it+1} = z_{it+1} \sum_{j=1}^{3} D_{ji} \left(e_{jt} + \frac{\phi+1}{\phi} R_j e_{jt} (1 - (\frac{e_{jt}}{K_j})^{\phi}) \right) \quad \forall i = 1, ..., 3$$

Theorem 2 characterizes the optimality conditions for the harvest game. Notably, these conditions are with known functional forms, facilitating the identification of the structural parameter θ_i via the GMM framework developed in Section 3.

Theorem 2 (Optimality Conditions for The Harvesting Game)**.** *Suppose Assumptions 1 and 2 hold.*[19] *Then, the interior MPNE* $(e_{1t}^*, ..., e_{Nt}^*)$ *is the fixed point of* N *optimality conditions:*

$$\mathbb{E}\left[-P + C + \delta D_{ii} z_{it+1} \left(1 + \frac{\phi+1}{\phi} R_i (1 - (\phi+1)(\frac{e_{it}}{K_i})^{\phi}) \right) \left(P - C + \frac{3\theta_i}{4 x_{it+1}^{0.25}} \right) | \mathcal{I}_t \right] = 0 \qquad (13)$$

for all $i = 1, ..., N$ *and* t*.*

Proof of Theorem 2: See Appendix. □

[19] Assumptions 3 and 4 automatically hold for this game.

4.3 Data

Item	Description	Source/value
	Panel of States, Controls, Shocks	
$\{x_{it}\}$	Fish Biomass in patch i in year t (unit: mt)	Costello et al. (2016), RAM Database
$\{e_{it}\}$	Escapement of Fish in patch i in year t (unit: mt)	Calculated from the public catch data
$\{z_{it}\}$	Recruitment Shock of Fish in patch i in year t	Inferred from the reproduction process
	Parameters of Payoff Function	
$P-C$	Marginal Profit	\$1041/mt (Costello et al., 2016)
δ	Annual Discount Factor	0.95
θ_i	Coefficient for Stock Value	To be estimated using GMM
	Parameters of Reproduction Dynamics	
D_{ij}	Fraction of fish migrates from patch i to j	$D_{ii}=0.95, D_{ij}=D_{ji}=0.025$
K_i	Carrying capacity of patch i	$K_{CN}=24794608, K_{JP}=2149800, K_{KR}=9238137$ (RAM and Costello et al., 2016)
R_i	Intrinsic growth rate of patch i	$R_{CN}=0.041, R_{JP}=0.065, R_{KR}=0.065$ (RAM and Costello et al., 2016)
ϕ	Curvature of the recruitment function	0.188 (Costello et al., 2016)

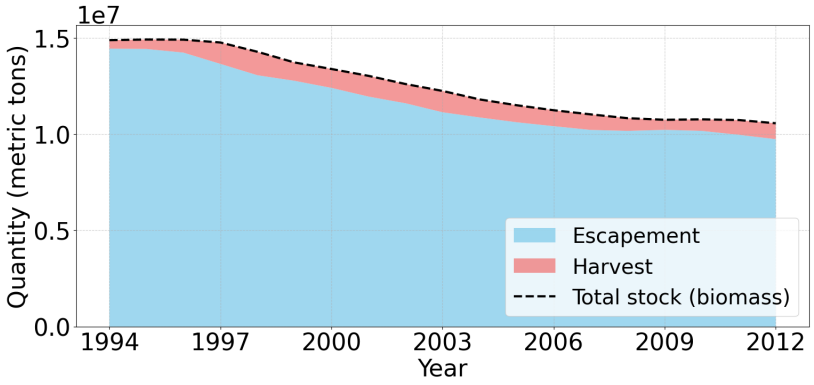
Table 1: Calibration Table.
Notes: This table reports the parameter values to be used in the GMM estimation below. The parameters to be estimated are θ_i for the three countries.

Our estimation primarily relies on the RAM Legacy Stock Assessment Database and the global fisheries dataset in Costello et al. (2016). The RAM Legacy Stock Assessment Database provides annual biomass estimates of Japanese anchovy for Japan from 1994 to 2012. The global fisheries dataset provides annual biomass estimates of Japanese anchovy for China and the Republic of Korea from 1994 to 2012.

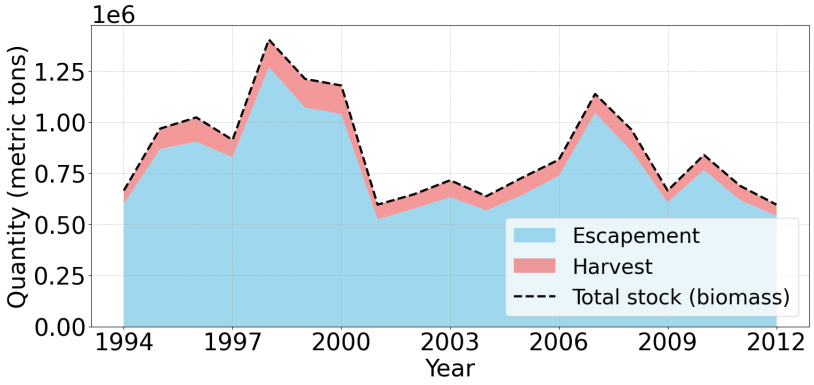
In addition, the two datasets include key biological parameters for the anchovy stocks, such as the carrying capacity, intrinsic growth rate, and the curvature of the recruitment function. Data on fish catch for each country are obtained from the Food and Agriculture Organization (FAO) of the United Nations. Table 1 summarizes the model's state variables, control variables, and parameters, along with their respective data sources.

Figure 3 visualizes the total stock of Japanese anchovy across the three countries and decomposes it into harvest and escapement. It shows that the scale of stock and harvest effort has been relatively stable over the 18-year period; China holds the largest biomass in its EEZ, followed by Korea. In our GMM estimation,

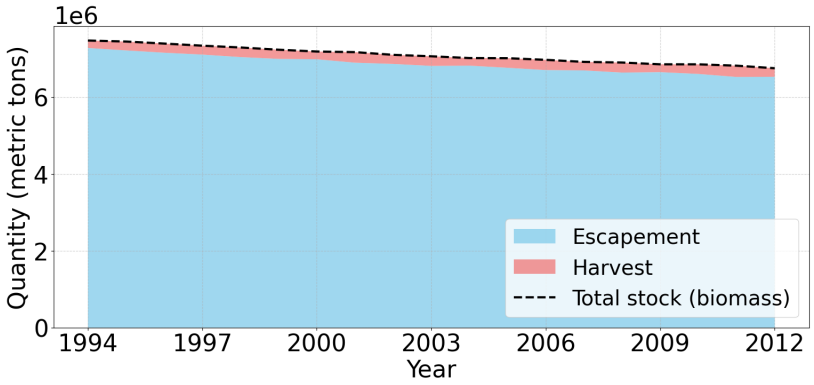
we set the annual discount factor to $\delta = 0.95$ and use a symmetric dispersal matrix with the same diagonal retention rate (i.e., $D_{ii} = D_{jj}$) and uniform off-diagonal dispersal rates (i.e, $D_{ij} = D_{ji}$). In our preferred specification, we set the retention rate to be $D_{ii} = 0.95$ and the dispersal rate to be $D_{ij} = D_{ji} = 0.025$, while we also report the estimation results with other plausible dispersal rates in the A.9. [20]



(a) China



(b) Japan



(c) Korea

Figure 3: Stacked Time Series of Annual Japanese Anchovy Harvest and Escapement, by Country.
Notes: Source: The RAM database and Costello et al. (2016). It shows that the scale of stock and harvest effort has been relatively stable over the 18-year period; China holds the largest biomass in its EEZ, followed by Korea.

[20] To improve the consistency of our analysis, we calibrate the reproduction function by matching the reproduction rate from the two different data sources. See the Appendix A.8 for more details.

4.4 GMM Estimation

Parameter	Coefficient	Std. Err.	z	p-value
θ for China	4641.5067	125.961	36.85	0.000
θ for Japan	27475.8417	2554.7181	10.75	0.000
θ for Korea	10871.4753	157.191	69.13	0.000

Table 2: GMM Estimates of The Stock Value Coefficients.
Notes: The units of the coefficients are denominated in 2017 USD per mt$^{0.75}$. The estimation is based on the retention rate = 0.95, and we also report the estimation results with other plausible dispersal rates in the A.9. The sample size is 54 (18 years of observation for 3 countries), and the model estimates 3 parameters. We use 15 moment conditions, derived by interacting the optimality condition with four instruments (x_{it-1}, x_{it-2}, e_{it-1}, e_{it-2}) and a constant, across all three countries. The standard errors reported are heteroskedasticity and autocorrelation robust (HAC) based on Bartlett kernels with 2 lags.

We estimate each country i's stock value parameter θ_i using optimality condition specified in equation 13. For each estimation, we employ two lags of country i's own biomass and escapement ($x_{it-1}, x_{it-2}, e_{it-1}, e_{it-2}$) as instrumental variables. Table 2 presents the estimates. The units of the coefficients are denominated in 2017 USD per mt$^{0.75}$, since ecological value is modeled as $\theta_i x_{it}^{0.75}$ with x_{it} in metric tons. A more interpretable magnitude is the marginal ecological value of stock. Evaluated at each country's average stock level, this equals \$59/mt for China, \$678/mt for Japan, and \$158/mt for Korea, against a marginal harvest profit of \$1,041/mt. Japan's EEZ manager therefore assigns the greatest value to leaving a marginal ton of anchovy in the water, and China's the least. We also report the results of GMM estimation using other values of the dispersal matrix in appendix A.9 and this pattern is consistent for different choices of D_{ii}.

The large z-statistics in Table 2 indicate that the estimated structural parameters are statistically significant from zero. However, these tests do not address whether the model is correctly specified. Each z-statistic is an asymptotic Wald test of the null hypothesis that the associated parameter equals zero. For the structural parameters of the stock value estimated here, this null is not the economically relevant hypothesis. Finding that the stock value of Anchovy is statistically distinguishable from zero is expected a priori and therefore does not, by itself, indicate that the model's moment conditions are valid. We therefore rely on Hansen's J-test as the model specification diagnostic.

To assess the overall fit and validity of our structural model, we report Hansen's J-statistic and the associated p-values in Table 3. In GMM estimation, the J-test serves as an omnibus test of the model's over-identifying restrictions (Hansen, 1982). The *null hypothesis* is that the population moment conditions are correctly specified and the instruments are exogenous. For all three countries, the p-values considerably exceed the 0.05 threshold, indicating that we fail to reject the null hypothesis. These results provide supplemental

evidence that our model is well-specified.

Parameter	Hansen's J-statistic (χ^2)	p-value
θ for China	4.3927	0.3554
θ for Japan	3.2166	0.5222
θ for Korea	3.9603	0.4114

Table 3: Hansen's J-test for Model Specification and Over-identifying Restrictions.
Notes: This table reports the results of Hansen's J-test, which serves as an omnibus test of the model's over-identifying restrictions and structural assumptions. Under the null hypothesis, the moment conditions are correctly specified and the instruments are exogenous. For all three countries, the p-values considerably exceed the 0.05 threshold. These results provide supplemental evidence that our model is well-specified.

Based on our GMM estimates in Table 2, we evaluate the relative contributions of harvest revenue and stock value to each EEZ manager's intertemporal utility. Figure 4 illustrates this welfare decomposition. Specifically, for China's EEZ manager, harvest profit constitutes approximately half of total intertemporal utility, whereas for the Japan's and the Republic of Korea's EEZ manager, it accounts for 10 percent and 15 percent. This result aligns with empirical patterns observed in the data, where China exhibits a substantially higher harvest rate than Japan and the Republic of Korea. Lastly, Japan places the greatest weight on stock value according to our GMM estimation, which is reflected by having the lowest ratio of harvest revenue to stock value in Figure 4.

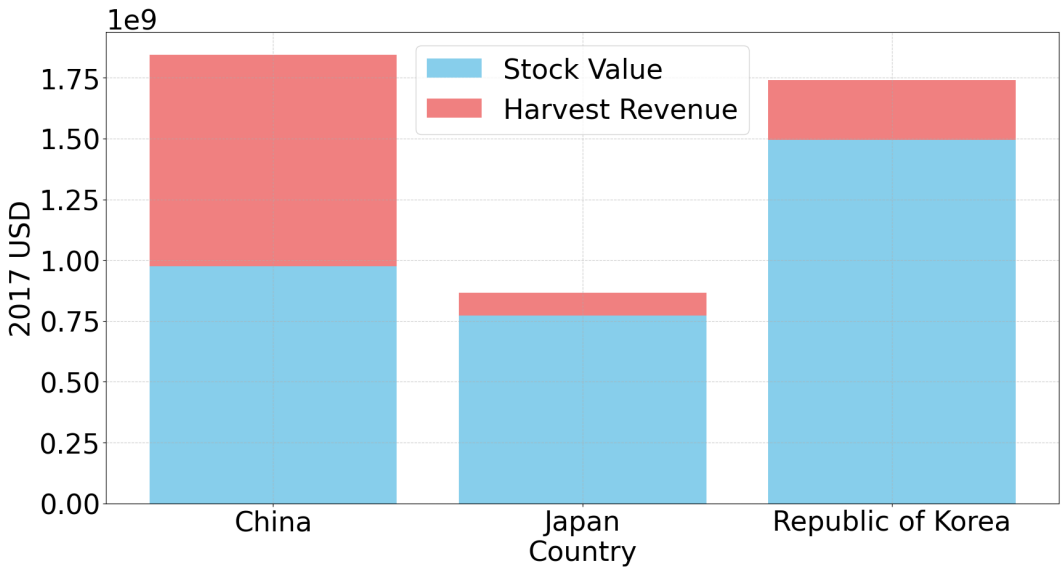
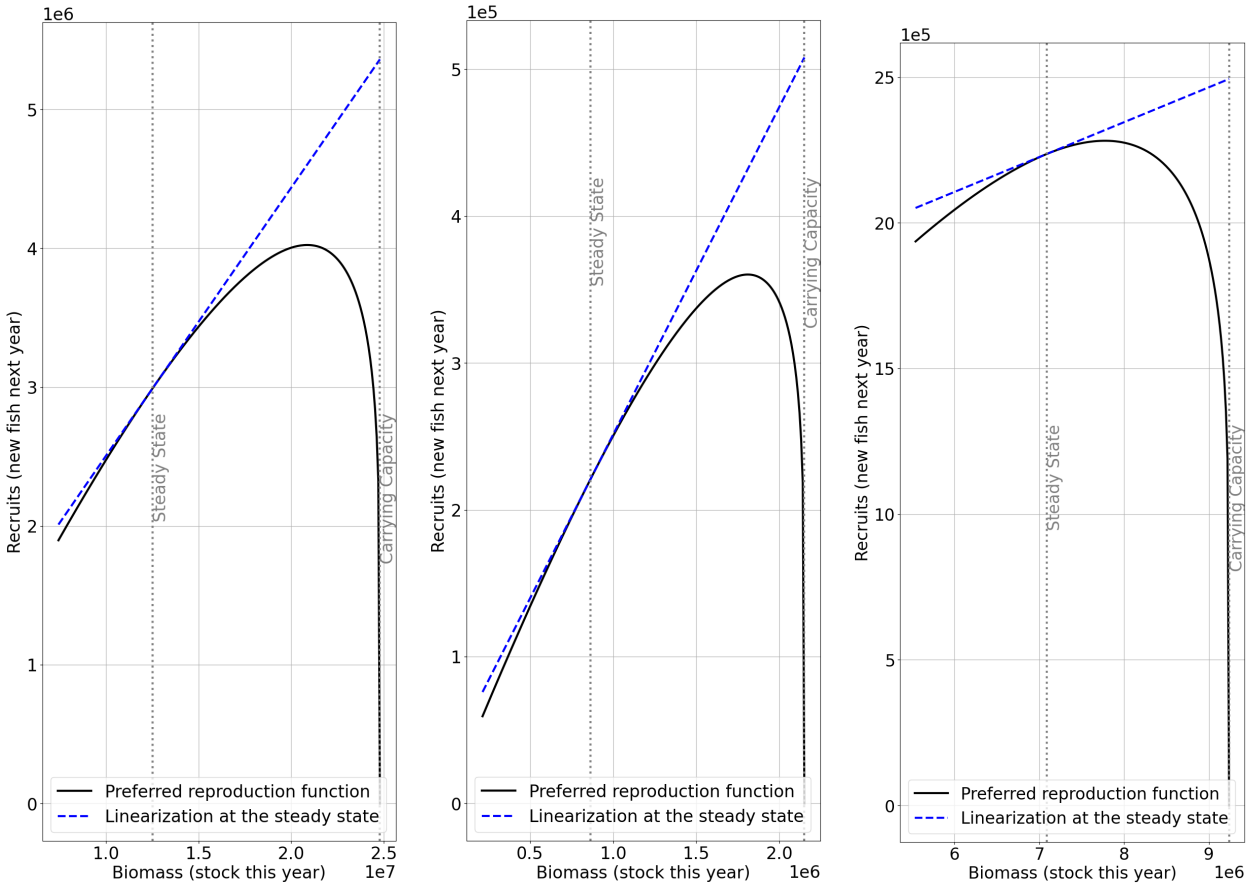


Figure 4: Decomposition of Welfare Derived from Japanese Anchovy, Annual Average of 1994-2012.
Notes: This welfare decomposition is based on the GMM estimates in Table 2. Specifically, Japan places the greatest weight on stock value according to our GMM estimation, which is reflected by having the lowest ratio of harvest revenue to stock value in Figure 4.

4.5 Accuracy Gains Over Linear Benchmarks

A key motivation for this paper is that nonlinearities in equations of motion—such as maximum sustainable yield in fish reproduction or exponential disease transmission—are profoundly *meaningful* for policymaking. Consequently, estimation approaches that rely on linearizing these dynamics around a steady state are inherently inadequate.

In this section, we illustrate the accuracy gains of our nonlinear approach using a simple thought experiment: if we were to linearize fish reproduction to apply traditional estimation methods (e.g., the linear-quadratic games), how biased would the resulting estimates be?[21]



(a) China (b) Japan (c) Republic of Korea

Figure 5: Japanese Anchovy Reproduction Dynamics: Nonlinear vs. Linearized.
Notes: The black lines represent the nonlinear reproduction dynamics (Equation 12) used for our primary GMM estimation. The blue dashed lines depict the reproduction functions linearized around each country's steady-state stock. These linearized functions serve as a misspecified benchmark for the alternative estimation results reported in Table 4.

[21] We acknowledge that fisheries scientists rarely linearize fish reproduction, as doing so violates fundamental biological principles. This linearization serves purely as a benchmark.

Figure 5 contrasts our nonlinear dynamics with this linear approximation benchmark. The solid black lines represent the nonlinear fish reproduction function used in our GMM estimation (Equation 12).[22] The dashed blue lines plot the reproduction functions *linearized* around each country's steady-state stock.

Parameter	Preferred Estimation (Nonlinear Dynamics)	Benchmark Estimation (Linearized Dynamics)
θ for China	4641.5067 (125.961)	-9720.7539 (116.592)
θ for Japan	27475.8417 (2554.7181)	15597.6153 (2191.4091)
θ for Korea	10871.4753 (157.191)	-1606.8876 (77.6586)

Table 4: Comparison of Stock Value Estimates: Nonlinear vs. Linearized Dynamics.
Notes: This table reports GMM estimates for the stock value parameter, θ_i. The preferred specification incorporates full nonlinear reproduction dynamics, while the benchmark uses functions linearized around the steady-state fish stock. Aside from this linearization, the estimation procedure remains identical to that in Table 2. Standard errors are in parentheses. The units of the coefficients are denominated in 2017 USD per $\text{mt}^{0.75}$.

Table 4 contrasts our stock value estimates with those from the misspecified linear benchmark. The results demonstrate that the linearized model considerably *underestimates* these stock values. Because the linear specification overstates reproduction speeds when stocks are low, it fundamentally understates the importance of *preventing stock depletion*. Consequently, we see a severe underestimation of how much countries value maintaining the stock level—a bias that can even push estimates negative for countries like China and Korea, which have greater curvature in their reproduction functions. These economically implausible negative estimates further underscore the necessity of incorporating nonlinearity into the model.

In summary, comparing our method to a linear benchmark illustrates substantial accuracy gains. Countries make strategic decisions based on nonlinear dynamics, meaning linear models will consistently misrepresent their incentives. This logic applies not only to fisheries but also broadly to fields like infectious disease management.

4.6 Equilibrium Effects of Shifting Migratory Patterns

Having estimated the country-specific parameters of stock valuation, we now assess the equilibrium impact of shifting migratory patterns using the harvesting game defined in Equation 12. Climate change is altering the migratory patterns and spatial distribution of numerous fish stocks, including the Japanese Anchovy (Lecomte et al., 2004; Ma et al., 2019). Liu et al. (2020) predict that the Japanese Anchovy will be more likely to shift northward in response to rising sea surface temperatures. Specifically, a greater proportion of

[22] Specifically, $y = \frac{\phi+1}{\phi} R_i x \left(1 - \left(\frac{x}{K_i}\right)^{\phi}\right)$ for each country i, where the y-axis is the recruits and the x-axis is stock.

the stock is expected to migrate into the Korean EEZ during the winter.

This shift raises a fundamental economic question: how will changes in migratory patterns affect harvest outcomes within each country's EEZ? Intuitively, one might hypothesize that increased migration of Japanese Anchovy to Korea would naturally lead to higher harvest levels in that jurisdiction.

However, we demonstrate that this hypothesis is not guaranteed to hold; the equilibrium effect of migratory patterns on harvest is ultimately an empirical question. This is because changes in migration affect not only the spatial distribution of biomass but also the strategic incentives of EEZ managers under a non-cooperative harvesting framework. Anticipating increased migratory flow into Korea, China and Japan may re-optimize their strategies by harvesting more intensely, recognizing that any stock they do not harvest is increasingly likely to migrate into Korean waters rather than remain available to them.

Such strategic responses require structural estimation and cannot be fully captured by reduced-form regressions or purely theoretical models. A reduced-form approach overlooks the forward-looking, strategic behavior central to this harvesting game. Conversely, a purely theoretical analysis yields ambiguous predictions regarding the effects of changes in biomass inflow, as illustrated by Theorem 3.[23]

Theorem 3 (Ambiguous Equilibrium Effects)**.** *Consider two agents, A and B, where migration patterns change exogenously from $\left[\begin{smallmatrix} D_{AA} & D_{AB} \\ D_{BA} & D_{BB} \end{smallmatrix}\right]$ to $\left[\begin{smallmatrix} D_{AA}-\Delta & D_{AB}+\Delta \\ D_{BA}-\Delta & D_{BB}+\Delta \end{smallmatrix}\right]$. Then, the sign of $\frac{dh_B^*}{d\Delta}$ is theoretically ambiguous, where h_B^* denotes the steady state equilibrium harvest of B.*[24]

Proof of Theorem 3: See Appendix. □

Theorem 3 implies that a stronger migration tendency toward Korea can lead to either an increase or a decrease in Korea's equilibrium harvest compared to the status quo. The intuition is that there are two opposing forces at play. First, knowing that the retention rate of Japanese Anchovy is higher in Korea, Korean managers have an incentive to increase escapement (holding China and Japan's strategies constant) to build stock, which will then increase long-run equilibrium harvest. Second, however, China and Japan—aware that the migration rate to Korea has increased—will respond by harvesting more intensely. This strategic reaction leaves less escapement available to migrate into Korea, thereby decreasing the stock available for Korea to harvest. The net equilibrium effect depends on the specific parameter values regarding migration strength, fish reproduction, and country-specific stock valuations. Theorem 3 shows that theory alone cannot

[23] This theorem builds on Costello et al. (2019), which examines the role of nonlinear harvest costs; in contrast, our analysis focuses on the role of nonlinear stock values.

[24] Here, $h_B^* \equiv x_B^* - e_B^*$.

determine which force dominates.

To quantify these effects, we simulate the dynamic game under altered migration patterns building on our GMM estimates. Figure 6 illustrates how a range of plausible migratory patterns change affects steady state harvest strategies, while Figure 7 displays the subsequent steady state stock of Japanese Anchovy within each country. In both figures, the X-axis represents the percentage increase in the migration flow into Korea, ranging from 0% (the status quo) to 200%. Under the status quo, $D_{ji} = 0.025$ (where $i \in \{\text{China, Japan}\}$ and $j = \text{Korea}$). In the 200% increase scenario, this rate rises to $D_{ji} = 0.075$.

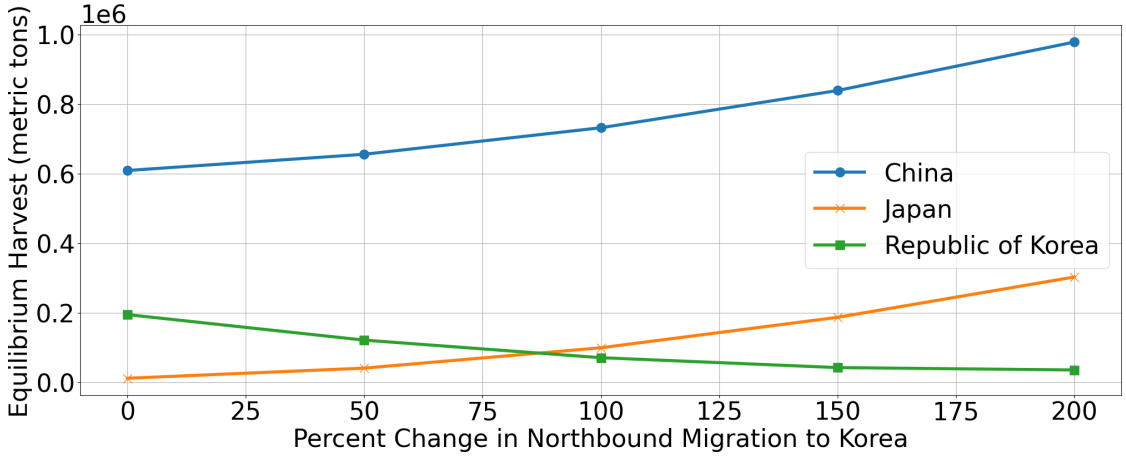


Figure 6: Steady State Annual Harvest of Japanese Anchovy after Changes in Migratory Patterns.
Notes: These results are derived from simulating the steady-state equilibrium of the game defined in Equation 12, using the parameter values in Table 1 and our GMM estimates in Table 2. The x-axis represents the percentage increase in migration flow into Korea. Under the status quo, $D_{ji} = 0.025$ (where $i \in$ China, Japan and $j =$ Korea). In the 200% increase scenario, this rate rises to $D_{ji} = 0.075$. The results show that as Anchovy migration into Korea increases, harvests by China and Japan rise, while Korea's harvest declines.

Two key results emerge from Figure 6. First, China and Japan increase their harvest. This is straightforward: if they do not harvest, there is now a higher probability that the saved fish will not be retained in their waters but will instead migrate to Korea. Knowing this, they increase harvest efforts.

Second, surprisingly, Korea has incentives to decrease their steady state harvest following the change in migration patterns. This result counters the intuition that Korea simply enjoy more harvest as migration inflow increases. Because Korea now enjoys higher retention and inflow, the private return from saving (i.e., not harvesting) fish increases. This private return comprises both the present value of future harvests and the ecological stock value—an unobservable parameter we estimated using GMM. In summary, this result highlights the necessity of modeling strategic responses via a multi-country dynamic game.

These harvest results are accompanied by changes in stock levels. Figure 7 verifies that Korea enjoys a

larger steady state stock, while China and Japan see reduced steady state stocks due to the shift in migration patterns. Note that these stock levels are evaluated at the new equilibrium harvest strategies.

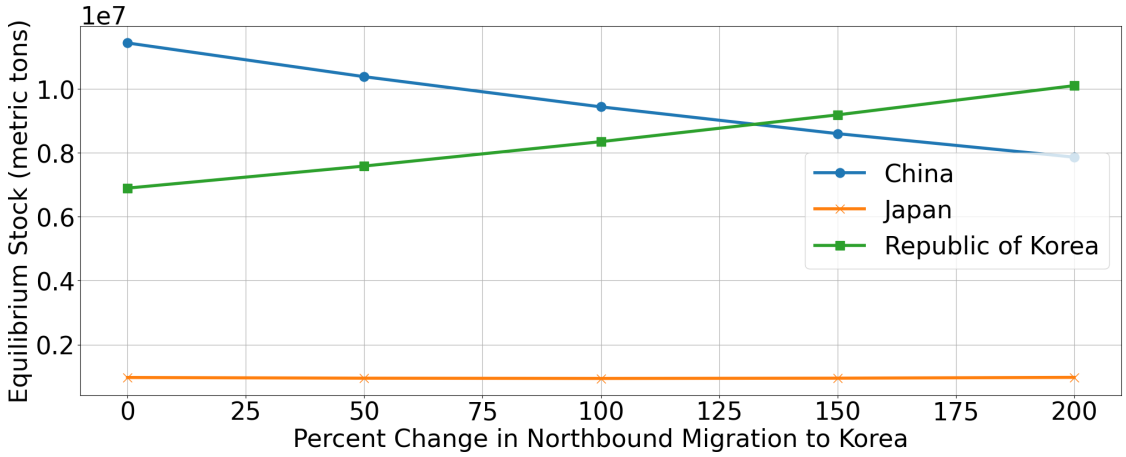
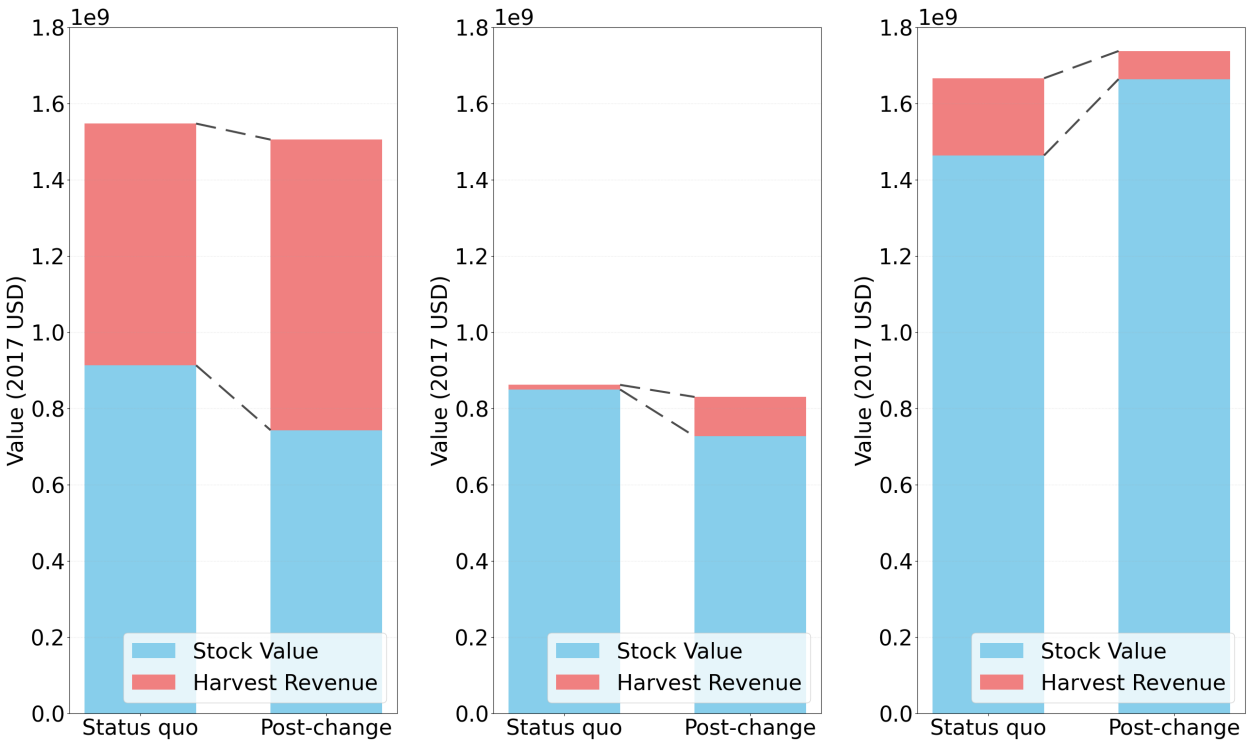


Figure 7: Steady State Stock Quantity of Japanese Anchovy after Changes in Migratory Patterns.
Notes: These results are derived from simulating the steady-state equilibrium of the game defined in Equation 12, using the parameter values in Table 1 and our GMM estimates in Table 2. The x-axis represents the percentage increase in migration flow into Korea. Under the status quo, $D_{ji} = 0.025$ (where $i \in$ China, Japan and $j =$ Korea). In the 200% increase scenario, this rate rises to $D_{ji} = 0.075$. The results show that as Anchovy migration into Korea increases, equilibrium stock in China and Japan decline, while Korea's stock increases.

Given the surprising results regarding harvest levels, we use our dynamic game to estimate welfare implications. It is important to note that a reduction in harvest for Korea does not translate into a reduction in welfare. Figure 8 decomposes the welfare before and after the migration shift for each country, which is evaluated at a 100% change in northbound migration to Korea.

We highlight two findings regarding the welfare implications. First, although China and Japan generate more harvest revenue in the post-change steady state (indicated by the increase in the red bars), they are worse off overall. This is due to significantly reduced stock retention, which lowers the stock value component of welfare (indicated in blue). Second, Korea benefits from the favorable change in migration patterns, but not because it harvests more. Instead, the benefit stems from higher stock retention within its EEZ, which generates significant stock value utility.



(a) China (b) Japan (c) Republic of Korea

Figure 8: Steady State Annual Welfare Decomposition.
Notes: These results are derived from the steady-state equilibrium in Figure 6 and Figure 7, focusing on a 100% change in northbound migration to Korea. The welfare quantification is based on the value functions in Equation 12 and the parameter values in Table 1 and 2. Korea benefits from the favorable change in migration patterns, but not because it harvests more. Instead, the benefit stems from higher stock retention within its EEZ, which generates significant stock value utility.

In summary, the equilibrium effects of shifting migration patterns cannot be derived from theory alone. While estimating a three-country game with nonlinear fish growth has historically been difficult, our GMM method provides a tractable estimation strategy. The results demonstrate that the equilibrium effects on harvest are economically significant and counter-intuitive. This GMM method can be widely applied to contexts where nonlinear biodynamics are prevalent, such as fisheries, disease management, or wildlife conservation, utilizing higher-quality data as it becomes available.

5 Conclusion

This paper contributes to the literature on dynamic resource games by developing a tractable structural estimation method using the Generalized Method of Moments (GMM). Our approach offers two key advantages. First, it preserves the full nonlinear structure of the equations of motion without sacrificing computational feasibility. Second, it accommodates the strategic interactions inherent in complex systems. Real-world

problems frequently involve multiple agents making interdependent decisions under nonlinear dynamics, ranging from neighboring countries harvesting shared migratory fish to interconnected cities managing infectious diseases.

While recent theoretical breakthroughs provide ways to analytically characterize these complex games, they frequently yield ambiguous predictions that are highly sensitive to underlying parameters. For example, as demonstrated in our application, theoretical models alone cannot determine whether a more favorable fish migration pattern would increase or decrease a country's steady state fish harvests due to opposing strategic incentives. This highlights exactly why it is necessary to recover the structural parameters of these models. Empirical estimation allows researchers to quantify unobserved structural parameters, such as the weights on fish stock's ecological value, thereby resolving theoretical ambiguities with data-driven evidence.

Looking ahead, our framework provides a tractable tool for empirical applications within resource economics and across broader disciplines. Within resource management, this GMM estimator can be naturally extended to problems such as common-pool groundwater extraction or the cross-border management of invasive species. In other fields where nonlinear dynamics are critical, such as public health, the framework is particularly relevant for modeling and evaluating epidemic control measures across interconnected populations. By providing a set of sufficient conditions and establishing the consistency and asymptotic normality of our estimator, we offer a statistically rigorous and tractable path for applying nonlinear dynamic games to urgent policy questions.

A Appendix

A.1 Proof of Remark 1

We begin by demonstrating that the dynamic envelope theorem is insufficient to facilitate GMM estimation in a game-theoretic setting. First, the first order conditions from the Bellman equations are:

$$\begin{aligned}\frac{\partial V_{it}(\mathbf{x}_t,\mathbf{z}_t)}{\partial u_{it}} &= \frac{\partial f(x_{it},u_{it},z_{it};\theta_i)}{\partial u_{it}} + \delta\mathbb{E}[\sum_{j=1}^{N}(\frac{\partial V_{it+1}(\mathbf{x}_{t+1},\mathbf{z}_{t+1})}{\partial x_{jt+1}}\frac{\partial x_{jt+1}}{\partial u_{it}} + \frac{\partial V_{it+1}(\mathbf{x}_{t+1},\mathbf{z}_{t+1})}{\partial z_{jt+1}}\frac{\partial z_{jt+1}}{\partial u_{it}})|\mathcal{I}_t] \\ &= \frac{\partial f(x_{it},u_{it},z_t;\theta_i)}{\partial u_{it}} + \delta\mathbb{E}[\sum_{j=1}^{N}\frac{\partial V_{it+1}(\mathbf{x}_{t+1},\mathbf{z}_{t+1})}{\partial x_{jt+1}}\frac{\partial x_{jt+1}}{\partial u_{it}}|\mathcal{I}_t] = 0\end{aligned} \tag{14}$$

for all i. The first equality follows from the chain rule, and the second arises from the Assumption 2, which implies $\frac{\partial z_{jt+1}}{\partial u_{it}} = 0$ for all i, j. If we have the analytical representations of $\frac{\partial V_{it}(\mathbf{x}_t,\mathbf{z}_t)}{\partial x_{jt}}$ for all $j = 1, ..., N$, we can lead them by one period to obtain $\frac{\partial V_{it+1}(\mathbf{x}_{t+1},\mathbf{z}_{t+1})}{\partial x_{jt+1}}$ and substitute these into Equation 14 to derive the analytical optimality conditions.

In single-agent dynamic programming problems, the dynamic envelope theorem is sufficient to obtain analytical representations of $\frac{\partial V_t(\mathbf{x}_t,\mathbf{z}_t)}{\partial x_t}$ (Acemoglu, 2008; Costello and Plantinga, 2023). However, in a dynamic game, the dynamic envelope theorem alone is insufficient. To formalize this, we rewrite the value function $V_{it}(\mathbf{x}_t, \mathbf{z}_t)$ for agent i at time t as the following Equation 15. This expression follows from the definition of a value function as the present value of payoffs, evaluated at the equilibrium control variables $\mathbf{u}_t^*(\mathbf{x}_t, \mathbf{z}_t)$:

$$V_{it}(\mathbf{x}_t,\mathbf{z}_t) = f(x_{it},u_{it}^*,z_{it};\theta_i) + \delta\mathbb{E}[V_{it+1}(\mathbf{x}_{t+1}^*,\mathbf{z}_{t+1})|\mathcal{I}_t] \tag{15}$$

Here, $x_{jt+1}^* = g_j(\mathbf{x}_t, \mathbf{u}_t^*, z_{t+1})$ for all j, evaluated at the equilibrium $\mathbf{u}_t^*(\mathbf{x}_t, \mathbf{z}_t)$. Differentiating both sides of Equation 15 with respect to x_{it} yields the following expression:

$$\begin{aligned}\frac{\partial V_{it}(\mathbf{x}_t,\mathbf{z}_t)}{\partial x_{it}} &= \frac{\partial f(x_{it},u_{it}^*,z_{it};\theta_i)}{\partial x_{it}} + \frac{\partial f(x_{it},u_{it}^*,z_{it};\theta_i)}{\partial u_{it}^*}\frac{\partial u_{it}^*}{\partial x_{it}} + \frac{\partial f(x_{it},u_{it}^*,z_{it};\theta_i)}{\partial z_{it}}\frac{\partial z_{it}}{\partial x_{it}} \\ &+ \delta\mathbb{E}[\sum_{j=1}^{N}(\frac{\partial V_{it+1}(\mathbf{x}_{t+1}^*,\mathbf{z}_{t+1})}{\partial x_{jt+1}^*}\frac{\mathrm{d}x_{jt+1}^*}{\mathrm{d}x_{it}} + \frac{\partial V_{it+1}(\mathbf{x}_{t+1}^*,\mathbf{z}_{t+1})}{\partial z_{jt+1}}\frac{\partial z_{jt+1}}{\partial x_{it}})|\mathcal{I}_t] \\ &= \frac{\partial f(x_{it},u_{it}^*,z_{it};\theta_i)}{\partial x_{it}} + \frac{\partial f(x_{it},u_{it}^*,z_{it};\theta_i)}{\partial u_{it}^*}\frac{\partial u_{it}^*}{\partial x_{it}} + \delta\mathbb{E}[\sum_{j=1}^{N}\frac{\partial V_{it+1}(\mathbf{x}_{t+1}^*,\mathbf{z}_{t+1})}{\partial x_{jt+1}^*}\frac{\mathrm{d}x_{jt+1}^*}{\mathrm{d}x_{it}}|\mathcal{I}_t] \\ &= \frac{\partial f(x_{it},u_{it}^*,z_{it};\theta_i)}{\partial x_{it}} + \frac{\partial f(x_{it},u_{it}^*,z_{it};\theta_i)}{\partial u_{it}^*}\frac{\partial u_{it}^*}{\partial x_{it}} \\ &+ \delta\mathbb{E}[\sum_{j=1}^{N}\frac{\partial V_{it+1}(\mathbf{x}_{t+1}^*,\mathbf{z}_{t+1})}{\partial x_{jt+1}^*}\sum_{k=1}^{N}(\frac{\partial x_{jt+1}^*}{\partial x_{it}} + \frac{\partial x_{jt+1}^*}{\partial u_{kt}^*}\frac{\partial u_{kt}^*}{\partial x_{it}})|\mathcal{I}_t]\end{aligned} \tag{16}$$

The first equality follows from the chain rule, and while the second arises from Assumption 2, which implies $\frac{\partial z_{it}}{\partial x_{it}} =$

0 and $\frac{\partial z_{jt+1}}{\partial x_{it}} = 0$). Finally, the third equality is obtained by applying the chain rule and noting that $x^*_{jt+1} = g_j(\mathbf{x}_t, \mathbf{u}^*_t, z_{it+1})$.

Lastly, applying the dynamic envelope theorem by substituting Equation 14 into the previous equation, we obtain:[25]

$$\frac{\partial V_{it}(\mathbf{x}_t, \mathbf{z}_t)}{\partial x_{it}} = \frac{\partial f(x_{it}, u^*_{it}, z_{it}; \theta_i)}{\partial x_{it}} + \delta \mathbb{E}[\sum_{j=1}^{N} \frac{\partial V_{it+1}(\mathbf{x}^*_{t+1}, \mathbf{z}_{t+1})}{\partial x^*_{jt+1}} (\frac{\partial x^*_{jt+1}}{\partial x_{it}} + \sum_{k \neq i} \frac{\partial x^*_{jt+1}}{\partial u^*_{kt}} \frac{\partial u^*_{kt}}{\partial x_{it}}) | \mathcal{I}_t] \tag{17}$$

We have shown that the expression for $\frac{\partial V_{it}(\mathbf{x}_t, \mathbf{z}_t)}{\partial x_{it}}$ still involves *value functions with unknown forms*. Therefore, we cannot derive an analytical form for $\frac{\partial V_{it+1}(\mathbf{x}_{t+1}, \mathbf{z}_{t+1})}{\partial x_{it+1}}$, which is a crucial step in obtaining the analytical Euler equation as discussed in Costello and Plantinga (2023). The intuition behind this result is that $\frac{\partial x^*_{it+1}}{\partial u_{jt}} \neq 0$ for $i \neq j$. In other words, each agent i does not have full control over their own x^*_{it+1} due to the interdependence in the equations of motion. Following a similar derivation as above, one can also show that there is not an analytical expression for $\frac{\partial V_{it}(\mathbf{x}_t, \mathbf{z}_t)}{\partial x_{jt}}$ for $j \neq i$ either. □

A.2 Proof of Lemma 1

In the proof of Remark 1, we showed that the optimality condition for each agent i is:

$$\frac{\partial f(x_{it}, u^*_{it}, z_{it}; \theta_i)}{\partial u^*_{it}} + \delta \mathbb{E}[\sum_{j=1}^{N} \frac{\partial V_{it+1}(\mathbf{x}^*_{t+1}, \mathbf{z}_{t+1})}{\partial x^*_{jt+1}} \frac{\partial x^*_{jt+1}}{\partial u^*_{it}} | \mathcal{I}_t] = 0 \tag{18}$$

By Assumption 4, $\frac{\partial f(x_{it}, u^*_{it}, z_{it}; \theta_i)}{\partial u^*_{it}}$ is independent of $\mathbf{x}_t$. Furthermore, Assumption 3 ensures that $\frac{\partial x^*_{jt+1}}{\partial u^*_{it}}$ is independent of $\mathbf{x}_t$ when evaluated at the equilibrium strategies $\mathbf{u}^*_t$. Lastly, $\frac{\partial V_{it+1}}{\partial x^*_{jt+1}}$ is also independent of $\mathbf{x}_t$ because by the definition of value functions, $V_{it+1}(\mathbf{x}^*_{t+1}, \mathbf{z}_{t+1})$ depends only on future states $\mathbf{x}^*_{t+1}$ and shocks $\mathbf{z}_{t+1}$.

Consequently, all terms in the optimality condition are independent of $\mathbf{x}_t$. As this independence holds for all agents $i = 1, ..., N$ and since the fixed point of this system of conditions fully characterizes the interior MPNE, Lemma 1 is established. □

A.3 Proof of Theorem 1

In Equation 17, we derived that:

$$\frac{\partial V_{it}(\mathbf{x}_t, \mathbf{z}_t)}{\partial x_{it}} = \frac{\partial f(x_{it}, u^*_{it}, z_{it}; \theta_i)}{\partial x_{it}} + \delta \mathbb{E}[\sum_{j=1}^{N} \frac{\partial V_{it+1}(\mathbf{x}^*_{t+1}, \mathbf{z}_{t+1})}{\partial x^*_{jt+1}} (\sum_{k=1}^{N} \frac{\partial x^*_{kt+1}}{\partial x_{it}} + \sum_{k \neq i} \frac{\partial x^*_{jt+1}}{\partial u^*_{kt}} \frac{\partial u^*_{kt}}{\partial x_{it}}) | \mathcal{I}_t] \tag{19}$$

Subsequently, given the state-independence of $\mathbf{u}^*_t$ established in Lemma 1, we have that $\frac{\partial u^*_{kt}}{\partial x_{it}} = 0$ for all i, k. That is, the equilibrium control variables (e.g., fish escapement) are independent of the initial state variables (e.g., fish stock).

[25] Specifically, Equation 14 implies $\left[\frac{\partial f(x_{it}, u^*_{it}, z_{it}; \theta_i)}{\partial u^*_{it}} + \delta \mathbb{E}[\sum_{j=1}^{N} \frac{\partial V_{it+1}(\mathbf{x}^*_{t+1}, \mathbf{z}_{t+1})}{\partial x^*_{jt+1}} \frac{\partial x^*_{jt+1}}{\partial u^*_{it}} | \mathcal{I}_t]\right] \frac{\partial u^*_{it}}{\partial x_{it}} = 0$

Moreover, $\frac{\partial x^*_{kt+1}}{\partial x_{it}} = \frac{\partial g_k(\mathbf{u}^*_t, z_{kt+1})}{\partial x_{it}} = 0$ for all i, k by Assumption 3 and Lemma 1. Consequently, $\frac{\partial V_{it}(\mathbf{x}_t, \mathbf{z}_t)}{\partial x_{it}}$ simplifies to:

$$\frac{\partial V_{it}(\mathbf{x}_t, \mathbf{z}_t)}{\partial x_{it}} = \frac{\partial f(x_{it}, u^*_{it}, z_{it}; \theta_i)}{\partial x_{it}} \tag{20}$$

With exactly the same steps, we can have that for $j \neq i$:[26]

$$\frac{\partial V_{it}(\mathbf{x}_t, \mathbf{z}_t)}{\partial x_{jt}} = 0 \tag{21}$$

By leading $\frac{\partial V_{it}}{\partial x_{it}} = \frac{\partial f(x_{it}, u^*_{it}, z_{it}; \theta_i)}{\partial x_{it}}$ and $\frac{\partial V_{it}}{\partial x_{jt}} = 0$ by one period, we have $\frac{\partial V_{it+1}}{\partial x^*_{it+1}} = \frac{\partial f(x^*_{it+1}, u^*_{it+1}, z_{it+1}; \theta_i)}{\partial x^*_{it+1}}$ and $\frac{\partial V_{it+1}}{\partial x^*_{jt+1}} = 0$. Substituting these into Equation 14 completes the proof of Theorem 1. □

A.4 Proof of Lemma 2

By law of iterated expectation and the optimality condition, we have that:

$$\begin{aligned}
\mathbb{E}[G_i(\mathbf{X}_t, \mathbf{U}_t, Z_t) r_i(w_t; \theta_i)] &= \mathbb{E}\left[\mathbb{E}[G_i(\mathbf{X}_t, \mathbf{U}_t, Z_t) r_i(\mathbf{w}_t; \theta_i) | \mathcal{I}_t]\right] \\
&= \mathbb{E}\left[G_i(\mathbf{X}_t, \mathbf{U}_t, Z_t) \mathbb{E}[r_i(\mathbf{w}_t; \theta_i) | \mathcal{I}_t]\right] \\
&= 0 \quad \square
\end{aligned} \tag{22}$$

A.5 Asymptotic Theory for the GMM Estimator

In this subsection, we establish the consistency and the asymptotic normality of our GMM estimator.

Assumption 5 (Conditions for Consistency)**.** *Assume that the following conditions hold:*

(i) (Identification) $\mathbb{E}[G_i(\mathbf{X}_t, \mathbf{U}_t, \mathbf{Z}_t) r_i(\mathbf{w}_t; \theta_i')] \neq 0$ *for all* $\theta_i' \neq \theta_i$.

(ii) The parameter space Θ *is a compact subset of* $\mathbb{R}^k$.

(iii) The time series of the states $\mathbf{x}_t$*, the control* $\mathbf{u}_t$*, and shocks* $\mathbf{z}_t$ *are strictly stationary and ergodic.*

(iv) The weighting matrix $\widehat{W}_{iT}$ *converges in probability to a positive definite matrix* W_i*, as* $T \to \infty$.

(v) For every $\mathbf{w}_t$*,* $r_i(\mathbf{w}_t, \theta)$ *is continuous in* θ.

(vi) $G_i(\mathbf{X}_t, \mathbf{U}_t, \mathbf{Z}_t)$ *is a measurable function of* $(\mathbf{X}_t, \mathbf{U}_t, \mathbf{Z}_t)$*, and* $r_i(\mathbf{w}_t, \theta)$ *is a measurable function of* $\mathbf{w}_t$ *for every* $\theta \in \Theta$.

(vii) (Dominance) $\mathbb{E}[\sup_{\theta \in \Theta} \|G_i(\mathbf{X}_t, \mathbf{U}_t, \mathbf{Z}_t) r_i(\mathbf{w}_t; \theta)\|] < \infty$[27].

Theorem 4 (Consistency)**.** *Under Assumption 5,* $\widehat{\theta}_i \xrightarrow{p} \theta_i$ *as* $T \to \infty$.

Proof of theorem 4: The results follows directly from Proposition 7.7 in Hayashi (2000). □

Assumption 6 (Conditions for Asymptotic Normality)**.** *Assume that the following conditions hold:*

[26] This step is an extension of the main result in Tsao and Costello (2024).

[27] All the norms in this section are the Euclidean norm.

(i) The true parameter θ_i lies in the interior of the parameter space Θ: $\theta_i \in int(\Theta)$.

(ii) For every $\mathbf{w}_t$, $r_i(\mathbf{w}_t;\theta)$ is continuously differentiable in θ.

(iii) $\mathbb{E}[G_i(\mathbf{X}_t,\mathbf{U}_t,\mathbf{Z}_t)G_i(\mathbf{X}_t,\mathbf{U}_t,\mathbf{Z}_t)'r_i(\mathbf{w}_t;\theta_i)^2] = \Sigma_i < \infty$.

(iv) (Local dominance) For some neighborhood $\mathcal{N}$ of θ_i, $\mathbb{E}\left[G_i(\mathbf{X}_t,\mathbf{U}_t,\mathbf{Z}_t)\sup_{\theta\in\mathcal{N}}\left\|\frac{\partial r_i(\mathbf{w}_t;\theta)}{\partial\theta'}\right\|\right] < \infty$.

(v) The matrix $R_i = \mathbb{E}\left[G_i(\mathbf{X}_t,\mathbf{U}_t,\mathbf{Z}_t)\frac{\partial r_i(\mathbf{w}_t;\theta)}{\partial\theta'}\right]$ *has full column rank.*

Theorem 5 (Asymptotic Normality and Covariance Matrix Estimation)**.** *Under Assumptions 5 and 6, the following results hold:*

(i) (Asymptotic normality) As $T\to\infty$, the GMM estimator $\widehat{\theta}_i$ is asymptotically normal with asymptotic covariance matrix

$$Avar(\widehat{\theta}_i) = (R_i'W_iR_i)^{-1}R_i'W_i\Sigma_iW_iR_i(R_i'W_iR_i)^{-1} \tag{23}$$

(ii) (Consistent estimation of the asymptotic covariance matrix) Suppose, in addition, that

$$\mathbb{E}\left[\|G_i(\mathbf{X}_t,\mathbf{U}_t,\mathbf{Z}_t)\|^2\sup_{\theta\in\mathcal{N}}\left\|r_i(\mathbf{w}_t,\theta)\frac{\partial r_i(\mathbf{w}_t,\theta)}{\partial\theta}\right\|\right] < \infty.$$

Define

$$\widehat{\Sigma}_i = \frac{1}{T}\sum_{t=1}^{T}G_i(\mathbf{X}_t,\mathbf{U}_t,\mathbf{Z}_t)G_i(\mathbf{X}_t,\mathbf{U}_t,\mathbf{Z}_t)'r_i(\mathbf{w}_t;\widehat{\theta}_i)^2$$

and

$$\widehat{R_i} = \frac{1}{T}\sum_{t=1}^{T}\left[G_i(\mathbf{X}_t,\mathbf{U}_t,\mathbf{Z}_t)\frac{\partial r_i(\mathbf{w}_t;\widehat{\theta}_i)}{\partial\theta'}.\right]$$

Then a consistent estimator of the asymptotic covariance matrix is:

$$\widehat{Avar(\widehat{\theta}_i)} = (\widehat{R}_i'\widehat{W}_i\widehat{R}_i)^{-1}\widehat{R}_i'\widehat{W}_i\widehat{\Sigma}_i\widehat{W}_i\widehat{R}_i(\widehat{R}_i'\widehat{W}_i\widehat{R}_i)^{-1} \tag{24}$$

Proof of theorem 5: It suffices to verify the conditions of Proposition 7.10 in Hayashi (2000). It is straightforward to show that conditions (i), (ii), (iv), and (v) in Assumption 6 are satisfied; thus, we focus on verifying condition (3) and the consistency of $\widehat{\Sigma}_i$. Regarding condition (iii), note that $\sigma(\mathbf{u}_t, z_t, \mathbf{u}_{t-1}, z_{t-1}, \dots) \subseteq \mathcal{I}_t$. Following the proof of Lemma 1 and the strict stationarity and ergodicity of $\{\mathbf{u}_t, z_t\}$, it follows that $G_i(\mathbf{U}_t, Z_t)r_i(\mathbf{w}_t;\theta_i)$ is a strictly stationary and ergodic martingale difference sequence (MDS) adapted to the filtration $\mathcal{I}_t$. By theorem 14.11 of Hansen (2022), $\frac{1}{\sqrt{T}}\sum_{t=1}^{T}G_i(\mathbf{U}_t,Z_t)r_i(\mathbf{w}_t;\theta_i) \xrightarrow{d} \mathcal{N}(0,\Sigma_i)$, which verifies condition (iii).

Lastly, we justify the consistency of $\widehat{\Sigma}_i$:

$$
\begin{aligned}
\left\|\widehat{\Sigma}_i - \Sigma_i\right\| &= \left\| \frac{1}{T}\sum_{t=1}^{T} G_i(\mathbf{U}_t, Z_t)G_i(\mathbf{U}_t, Z_t)' r_i(\mathbf{w}_t, \widehat{\theta}_i)^2 - \mathbb{E}\left[G_i(\mathbf{U}_t, Z_t)G_i(\mathbf{U}_t, Z_t)' r_i(\mathbf{w}_t, \theta_i)^2\right] \right\| \\
&\leq \left\| \frac{1}{T}\sum_{t=1}^{T} G_i(\mathbf{U}_t, Z_t)G_i(\mathbf{U}_t, Z_t)' r_i(\mathbf{w}_t, \widehat{\theta}_i)^2 - \frac{1}{T}\sum_{t=1}^{T} G_i(\mathbf{U}_t, Z_t)G_i(\mathbf{U}_t, Z_t)' r_i(\mathbf{w}_t, \theta_i)^2 \right\| \\
&+ \left\| \frac{1}{T}\sum_{t=1}^{T} G_i(\mathbf{U}_t, Z_t)G_i(\mathbf{U}_t, Z_t)' r_i(\mathbf{w}_t, \theta_i)^2 - \mathbb{E}\left[G_i(\mathbf{U}_t, Z_t)G_i(\mathbf{U}_t, Z_t)' r_i(\mathbf{w}_t, \theta_i)^2\right] \right\| \\
&\leq \left\| \frac{1}{T}\sum_{t=1}^{T} G_i(\mathbf{U}_t, Z_t)G_i(\mathbf{U}_t, Z_t)' [r_i(\mathbf{w}_t, \widehat{\theta}_i)^2 - r_i(\mathbf{w}_t, \theta_i)^2] \right\| + o_p(1) \\
&= \left\| \frac{1}{T}\sum_{t=1}^{T} 2G_i(\mathbf{U}_t, Z_t)G_i(\mathbf{U}_t, Z_t)' r_i(\mathbf{w}_t, \bar{\theta}_i) \frac{\partial r_i(\mathbf{w}_t, \bar{\theta}_i)}{\partial \theta} (\widehat{\theta}_i - \theta_i) \right\| + o_p(1) \\
&\leq \left[\frac{1}{T}\sum_{t=1}^{T} 2 \left\|G_i(\mathbf{U}_t, Z_t)\right\|^2 \sup_{\theta \in \mathcal{N}} \left\| r_i(\mathbf{w}_t, \theta) \frac{\partial r_i(\mathbf{w}_t, \theta)}{\partial \theta} \right\| \right] \left\|\widehat{\theta}_i - \theta_i\right\| + o_p(1) \\
&\xrightarrow{p} 2\mathbb{E}\left[\left\|G_i(\mathbf{U_t}, Z_t)\right\|^2 \sup_{\theta \in \mathcal{N}} \left\| r_i(\mathbf{w}_t, \theta) \frac{\partial r_i(\mathbf{w}_t, \theta)}{\partial \theta} \right\| \right] o_p(1) + o_p(1) \\
&= o_p(1)
\end{aligned}
$$

where the third inequality follows from the fact that $G_i(\mathbf{U}_t, Z_t)G_i(\mathbf{U}_t, Z_t)' r_i(\mathbf{w}_t; \theta_i)^2$ is strictly stationary and ergodic. Thus, by the Ergodic Theorem (Theorem 14.9 in Hansen, 2022), the sample mean converges to its expectation. The fourth equality follows from the Mean Value Theorem. The second last convergence follows from the ergodic theorem and the fact that $\widehat{\theta}_i \xrightarrow{p} \theta_i$. The last equality follow from the condition imposed in Theorem 5 (ii). □

A.6 Proof of Theorem 2

Theorem 2 follows directly from applying the specific functional forms of the fishery model to Theorem 1. In Theorem 1, we established that the optimality conditions are given by:

$$
\mathbb{E}\left[\frac{\partial f(x_{it}, e_{it}, z_{it}; \theta_i)}{\partial e_{it}} + \delta \frac{\partial x_{it+1}}{\partial e_{it}} \frac{\partial f(x_{it+1}, e_{it+1}, z_{it+1}; \theta_i)}{\partial x_{it+1}} | \mathcal{I}_t \right] = 0 \tag{25}
$$

for all i and t. In the fisheries model, the functional forms give $\frac{\partial f(x_{it}, e_{it}, z_{it}; \theta_i)}{\partial e_{it}} = -(P - C)$, $\frac{\partial f(x_{it+1}, e_{it+1}, z_{it+1}; \theta_i)}{\partial x_{it+1}} = (P - C) + b'(x_{it+1}; \theta_i)$, and $\frac{\partial x_{it+1}}{\partial e_{it}} = D_{ii} z_{it+1} \left(1 + \frac{\phi+1}{\phi} R_i (1 - (\phi + 1)(\frac{e_{it}}{K_i})^{\phi})\right)$ by taking the derivative. □

A.7 Proof of Theorem 3

For clarity, we consider the deterministic case without stochastic shocks z_t; the results extend naturally to the stochastic case. We focus on the steady state equilibrium escapement (e_A^*, e_B^*) following a shift in the migration pattern Δ. The

steady state is characterized by the optimality conditions for regions A and B:

$$-P+C+\delta(D_{AA}-\Delta)\left(1+\frac{\phi+1}{\phi}R_A(1-(\phi+1)(\frac{e_A^*}{K_A})^{\phi})\right)\left(P-C+\frac{3\theta_A}{4{x_A^*}^{0.25}}\right)=0 \tag{26}$$

$$-P+C+\delta(D_{BB}+\Delta)\left(1+\frac{\phi+1}{\phi}R_B(1-(\phi+1)(\frac{e_B^*}{K_B})^{\phi})\right)\left(P-C+\frac{3\theta_B}{4{x_B^*}^{0.25}}\right)=0 \tag{27}$$

subject to the equations of motion for the transition stocks:

$$x_A^*=(D_{AA}-\Delta)\left(e_A^*+\frac{\phi+1}{\phi}R_Ae_A^*(1-(\frac{e_A^*}{K_A})^{\phi})\right)+(D_{BA}-\Delta)\left(e_B^*+\frac{\phi+1}{\phi}R_Be_B^*(1-(\frac{e_B^*}{K_B})^{\phi})\right) \tag{28}$$

$$x_B^*=(D_{AB}+\Delta)\left(e_A^*+\frac{\phi+1}{\phi}R_Ae_A^*(1-(\frac{e_A^*}{K_A})^{\phi})\right)+(D_{BB}+\Delta)\left(e_B^*+\frac{\phi+1}{\phi}R_Be_B^*(1-(\frac{e_B^*}{K_B})^{\phi})\right) \tag{29}$$

Note that because of the inter-regional migration flow, the steady state of region B depends strategically on the escapement choice of region A. Here, the time subscripts are dropped since we focus on the steady state.

We examine how the equilibrium harvest in region B, $h_B^*=x_B^*-e_B^*$, responds to a change in the migration parameter Δ. Applying the chain rule:

$$\begin{aligned}\frac{\mathrm{d}h_B^*(e_A^*,e_B^*)}{\mathrm{d}\Delta}&=\frac{\mathrm{d}x_B^*(e_A^*,e_B^*)}{\mathrm{d}\Delta}-\frac{\mathrm{d}e_B^*}{\mathrm{d}\Delta}\\&=(\frac{\mathrm{d}x_B^*(e_A^*,e_B^*)}{\mathrm{d}e_A^*}\frac{\mathrm{d}e_A^*}{\mathrm{d}\Delta}+\frac{\mathrm{d}x_B^*(e_A^*,e_B^*)}{\mathrm{d}e_B^*}\frac{\mathrm{d}e_B^*}{\mathrm{d}\Delta})-\frac{\mathrm{d}e_B^*}{\mathrm{d}\Delta}\\&=[(D_{AB}+\Delta)g_A'(e_A^*)]\frac{\mathrm{d}e_A^*}{\mathrm{d}\Delta}+[(D_{BB}+\Delta)g_B'(e_B^*)-1]\frac{\mathrm{d}e_B^*}{\mathrm{d}\Delta}\end{aligned} \tag{30}$$

where $g_i(\cdot)$ denotes $\left(e_i^*+\frac{\phi+1}{\phi}R_ie_i^*(1-(\frac{e_i^*}{K_i})^{\phi})\right)$, the post-growth population before migration.

Even under the assumption that $\frac{\mathrm{d}e_A^*}{\mathrm{d}\Delta}<0$ and $\frac{\mathrm{d}e_B^*}{\mathrm{d}\Delta}>0$, the weighted sum of these derivatives does not yield a monotonic sign without further parametric restrictions.[28] □

A.8 Details of Data Processing

Given escapement e_{jt}, the number of new fish reproduced in country j before dispersal is:

$$\frac{\phi+1}{\phi}R_je_{jt}(1-(\frac{e_{jt}}{K_j})^{\phi})$$

We match the growth rate parameter R_j and the biomass parameters K_{jt}, x_{jt} from RAM dataset (Japan) with Global Fisheries dataset (China and Republic of Korea), while keeping both (1) the time series of new fish reproduced and (2) the time series of harvest unchanged.

[28] The explicit expressions for $\frac{\mathrm{d}e_A^*}{\mathrm{d}\Delta}$ and $\frac{\mathrm{d}e_B^*}{\mathrm{d}\Delta}$ can be derived via Cramer's Rule applied to the linearized system of optimality conditions.

Specifically, we set $R'_{JP} = \max\{R_{CN}, R_{KR}\}$ and scale up the biomass metrics (K_{JP} and $\{x_{JP,t}\}$), while holding the number of new fish reproduced constant:

$$\frac{\phi+1}{\phi} R_{JP}(\bar{x}_{JP} - \bar{h}_{JP})(1 - (\frac{\bar{x}_{JP} - \bar{h}_{JP}}{K_{JP}})^{\phi}) = \frac{\phi+1}{\phi} R'_{JP}(\bar{x}'_{JP} - \bar{h}_{JP})(1 - (\frac{\bar{x}'_{JP} - \bar{h}_{JP}}{K'_{JP}})^{\phi})$$

On the left-hand side, $\bar{x}_{JP}$ and $\bar{h}_{JP}$ denote the average biomass inferred from the RAM dataset and the harvest observed in the RAM dataset. On the right-hand side, $\bar{x}'_{JP}$, r'_{JP}, and K'_{JP} are the re-calibrated parameter values consistent with the global fisheries dataset.

This matching procedure preserves the time series of both harvest and reproduction, yielding the following parameters: $R_{JP} = 0.065$ and $K_{JP} = 2,149,800$.

A.9 GMM Results for Various Dispersal Parameter Values

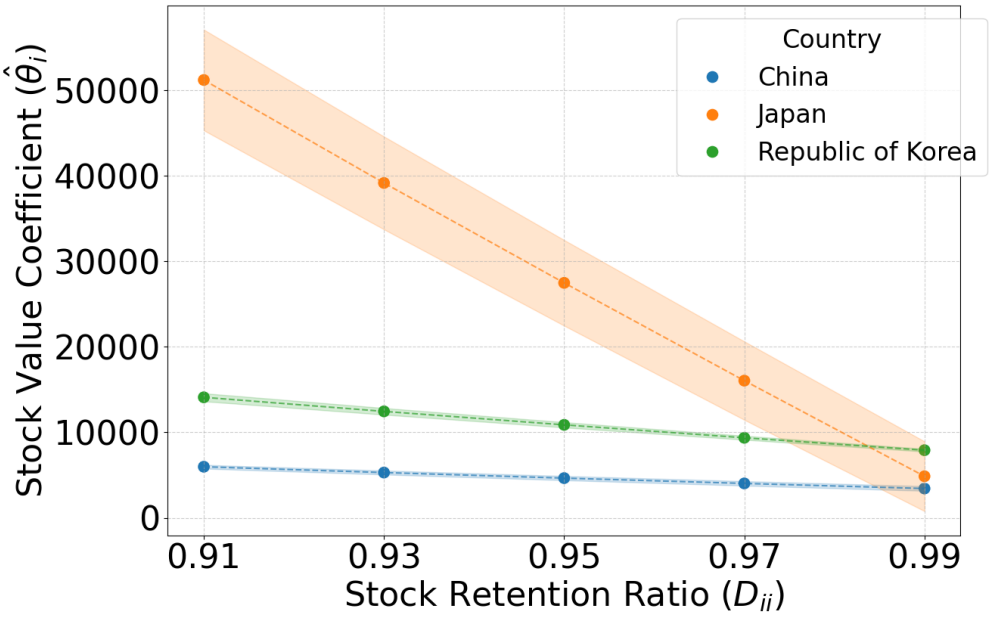


Figure 9: GMM-Estimated Stock Value Parameters.
Notes: GMM estimation results using dispersal parameters other than the baseline value of $D_{ii} = 0.95$. These results should be compared with the coefficients presented in Table 2. The units of the coefficients are denominated in 2017 USD per $\text{mt}^{0.75}$.

This appendix reports the GMM estimation results using dispersal parameters other than the baseline value of $D_{ii} = 0.95$. These results should be compared with the coefficients presented in Table 2. As shown in the figure, the ordering of magnitudes among the three countries remains consistent across different dispersal parameter values.

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